Multiplex Information Networks for Spatially Evolving Multiagent Formations*

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Abstract— **Current distributed control methods have a lack of information exchange infrastructure to enable spatially evolving multiagent formations. These methods are designed based on information exchange rules for a single layer network, which leads to multiagent formations with fixed, non-evolving spatial properties. For situations where capable agents have to control the resulting formation through these methods, they can only do so if such vehicles have global information exchange ability; however, this is not practical for cases that have large numbers of agents and low-bandwidth peer-to-peer communications.**

The contribution of this paper is to show how information exchange rules, which are represented by a network having multiple layers (multiplex information networks), can be designed for enabling spatially evolving multiagent formations. Specifically, we consider the formation tracking problem and introduce a novel distributed control architecture that allows capable agents to spatially alter density and orientation of the resulting formation while tracking a dynamic, non-stationary target without requiring global information exchange ability. In addition, we use tools and methods from differential potential fields to generalize the proposed architecture to allow for connectivity maintenance and collision avoidance that are needed in real-world applications. Stability of the proposed approaches is theoretically analyzed and their efficacy are illustrated on a numerical example.

I. Introduction

As advances in VLSI and MEMS technologies have boosted the development of integrated microsystems that combine mobility, computing, communication, and sensing on a single platform, future military and civilian operations will have the capability to exploit large numbers of interconnected agents such as low-cost and small-in-size autonomous vehicles and microsensors. Such large-scale multiagent systems will support operations ranging from environment monitoring and military surveillance, to guidance, navigation, and control of autonomous underwater, ground, aerial, and space vehicles.

For performing operations with dramatically increasing levels of complexity, multiagent systems require advanced distributed information exchange rules in order to make these systems evolve spatially for adapting dynamic environments and effectively responding to human interventions. Yet, current distributed control methods lack information exchange infrastructures to enable spatially evolving multiagent formations. This is due to the fact that these methods are designed based on information exchange rules for a network having a single layer (see, for example, [1]–[3] and references therein), which leads to multiagent formations with fixed, non-evolving spatial properties. For situations where capable agents have to control the resulting formation through these methods, they can only do so if such vehicles have global information exchange ability, but this is not practical for cases with large numbers of agents and low-bandwidth peer-to-peer communications.

A. Contribution

The contribution of this paper is to show how information exchange rules that are represented by a network with multiple layers (multiplex information networks) can be designed for enabling spatially evolving multiagent formations. Specifically, we consider the formation tracking problem and introduce a novel distributed control architecture that allows capable agents to spatially alter density and orientation of the resulting formation while tracking a dynamic, non-stationary target without requiring global information exchange ability. In addition, we use tools and methods from differential potential fields to generalize the proposed architecture to allow connectivity maintenance and collision avoidance that are needed in real-world applications. Stability of the proposed approaches is theoretically analyzed and their efficacy are illustrated on a numerical example.

B. Literature Review

Studies on multiplex information networks have recently emerged in the physics and networks science literatures, where they consider system-theoretic characteristics of network dynamics with multiple layers subject to intralayer and interlayer information exchange [4]–[11] (there also exist studies on multiplex networks that do not consider system-theoretic characteristics; see [12] for an excellent survey on this topic). However, these studies mainly consider cases where all layers perform simple consensus algorithms and analyze the convergence of the overall multiagent systems in the presence of not only intralayer but also interlayer information exchange, and hence, they do not deal with controlling spatial properties of multiagent formations. Note that there are also recent studies on networks of networks by the authors of [13]–[15]. However, these studies deal with

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large-scale systems formed from smaller factor networks via graph Cartesian products; hence, they are also not related with the contribution of this paper.

Spatial multiagent formation control is considered by the authors of [16]–[19] using approaches different from multiplex information networks. In particular, the authors of [16]–[18] assume that some of the formation design parameters are known globally by all agents, and the authors of [19] assume global knowledge of the complete network at the analysis stage. However, as previously discussed, such assumptions may not be practical in the presence of large numbers of agents and low-bandwidth peer-to-peer communications. From a data security point of view, in addition, it should be noted that one may not desire a multiagent system with all agents sharing some global information about an operation of interest. Throughout this paper, we do not make such assumptions in our multiplex information networks-based spatial multiagent formation control approach.

II. Notation and Mathematical Preliminaries

In this section, we introduce the notation used throughout the paper and recall some basic notions from graph theory, which are followed by the general setup of consensus and formation problems for multiagent systems that are necessary to establish the main results of this paper. For additional details about graph theory and multiagent systems, we refer to the excellent textbooks [1], [2], [20].

A. Notation

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrices, $\mathbb{R}_+$ (resp. $\overline{\mathbb{R}}_+$) denotes the set of positive (resp. non-negative-definite) real numbers, $\mathbb{R}_+^{n\times n}$ (resp. $\overline{\mathbb{R}}_+^{n\times n}$) denotes the set of $n \times n$ positive-definite (resp. non-negative-definite) real matrices, $\mathbb{S}_+^{n\times n}$ (resp. $\overline{\mathbb{S}}_+^{n\times n}$) denotes the set of $n\times n$ symmetric positive-definite (resp. symmetric nonnegative-definite) real matrices, $\mathbf{0}_n$ denotes the $n \times 1$ vector of all zeros, $\mathbf{1}_n$ denotes the $n \times 1$ vector of all ones, $\mathbf{0}_{n\times n}$ denotes the $n \times n$ zero matrix, and I_n denotes the $n\times n$ identity matrix. In addition, we write $(\cdot)^{\mathrm{T}}$ for transpose, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ for the minimum and maximum eigenvalue of the Hermitian matrix A, respectively; $\lambda_i(A)$ for the *i*-th eigenvalue of A, where A is symmetric and the eigenvalues are ordered from least to greatest value, $\det(A)$ for the determinant of A, $\mathrm{diag}(a)$ for the diagonal matrix with the vector a on its diagonal, $[x]_i$ for the entry of the vector x on the *i*-th row, and $[A]_{ij}$ for the entry of the of the matrix A on the *i*-th row and *j*-th column.

B. Notions from Graph Theory

In the multiagent systems literature, graphs are broadly adopted to encode interactions between networked agents. An undirected graph $\mathcal{G}$ is defined by a set $\mathcal{V}_\mathcal{G} = \{1, \ldots, n\}$ of nodes and a set $\mathcal{E}_\mathcal{G} \subset \mathcal{V}_\mathcal{G} \times \mathcal{V}_\mathcal{G}$ of edges. If the distance between two arbitrary nodes is less than R, then they are said to be neighbors and the neighboring relation is denoted by $j \in \mathcal{N}_i \triangleq \{j|\ j \in \mathcal{V}_\mathcal{G}, \|x_{ij}\|_2 < R\}$, where $x_{ij} \triangleq x_i - x_j$ with x_i and x_j being the state (position) of nodes i and j, respectively. In addition, if $(i, j) \in \mathcal{E}_\mathcal{G}$, then the nodes i and j are said to be formation neighbors [21], [22] and this relation is denoted by $j \in \mathcal{N}_i^f$, where $\mathcal{N}_i^f$ is a subset of $\mathcal{N}_i$. In general, note that $\mathcal{N}_i$ can be a time-varying set while $\mathcal{N}_i^f$ is a static set, that is, $\mathcal{N}_i^f$ remains unchanged in the presence of node movements. The degree of a node is given by the number of its formation neighbors. In particular, letting d_i be the degree of node i, the degree matrix of a graph $\mathcal{G}$, $\mathcal{D}(\mathcal{G}) \in \mathbb{R}^{n\times n}$, is given by

$$\mathcal{D}(\mathcal{G}) \triangleq \mathrm{diag}(d), \quad d = [d_1, \ldots, d_n]^{\mathrm{T}}. \tag{1}$$

A path $i_0 i_1 \ldots i_L$ is a finite sequence of nodes such that $i_{k-1} \in \mathcal{N}_{i_k}^f$ with $k = 1, \ldots, L$, and a graph $\mathcal{G}$ is connected if there exists a path between any pair of distinct nodes. The adjacency matrix of a graph $\mathcal{G}$, $\mathcal{A}(\mathcal{G}) \in \mathbb{R}^{n\times n}$, is given by

$$[\mathcal{A}(\mathcal{G})]_{ij} \triangleq \begin{cases} 1, & \text{if } (i,j) \in \mathcal{E}_\mathcal{G}, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The Laplacian matrix of a graph, $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_+^{n\times n}$, which plays a central role in many graph-theoretic treatments of multiagent systems, is given by

$$\mathcal{L}(\mathcal{G}) \triangleq \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G}), \tag{3}$$

where the spectrum of the Laplacian for an undirected and connected graph $\mathcal{G}$ can be ordered as

$$0 = \lambda_1(\mathcal{L}(\mathcal{G})) < \lambda_2(\mathcal{L}(\mathcal{G})) \le \cdots \le \lambda_n(\mathcal{L}(\mathcal{G})), \tag{4}$$

with $\mathbf{1}_n$ as the eigenvector corresponding to the zero eigenvalue $\lambda_1(\mathcal{L}(\mathcal{G}))$ and $\mathcal{L}(\mathcal{G})\mathbf{1}_n = \mathbf{0}_n$ and $\mathrm{e}^{\mathcal{L}(\mathcal{G})}\mathbf{1}_n = \mathbf{1}_n$ hold. In this paper, we assume that the graph $\mathcal{G}$ is undirected and connected unless noted otherwise.

C. Consensus Dynamics

We can model a given multiagent system by a graph $\mathcal{G}$, where nodes and edges represent agents and interagent information exchange links, respectively. Let $x_i(t) \in \mathbb{R}^m$ denote the state of node i, whose dynamics is described by the single integrator

$$\dot{x}_i(t) = u_i(t), \quad x_i(0) = x_{i0}, \quad i = 1, \cdots, n, \tag{5}$$

with $u_i(t) \in \mathbb{R}^m$ being the control input of node i. Allowing agent i to have access to the relative state information with respect to its formation neighbors, a solution to the consensus problem can be achieved, for example, by applying

$$u_i(t) = -\sum_{j \in \mathcal{N}_i^f} \big(x_i(t) - x_j(t)\big), \tag{6}$$

to the single integrator dynamics given by (5) [1], [2], where (5) in conjunction with (6) can be represented as the Laplacian dynamics of the form

$$\dot{x}(t) = -\mathcal{L}(\mathcal{G}) \otimes \mathrm{I}_m\, x(t), \quad x(0) = x_0, \tag{7}$$

with $x(t) = [x_1^{\mathrm{T}}(t), \cdots, x_n^{\mathrm{T}}(t)]^{\mathrm{T}}$ denoting the aggregated state vector of the multiagent system. Since the graph $\mathcal{G}$ is assumed to be undirected and connected, it follows from (7) that

$$\lim_{t\to\infty}[x_i(t)]_j = \frac{[x_1(0)]_j + \cdots [x_n(0)]_j}{n}, \quad (8)$$

holds for $i = 1, \ldots, n$ and $j = 1, \ldots, m$. In this paper, we assume that $m = 2$ without loss of generality, which implies that the multiagent system evolves in a planar space.

D. Formation Dynamics

For our take on the formation problem, define $\tau_i(t) \in \mathbb{R}^2$ as the displacement of $x_i(t) \in \mathbb{R}^2$ from the desired formation position of agent i, $\xi_i \in \mathbb{R}^2$. Using the state transformation given by

$$\tau_i(t) = x_i(t) - \xi_i, \quad i = 1, \ldots, n, \quad (9)$$

a solution to the formation problem follows from (7) with $m = 2$ as

$$\dot{x}(t) = -\mathcal{L}(\mathcal{G}) \otimes \mathrm{I}_2\, x(t) + \mathcal{L}(\mathcal{G}) \otimes \mathrm{I}_2\, \xi, \quad x(0) = x_0, \quad (10)$$

[1], [2], where $\xi = [\xi_1, \cdots, \xi_n]^{\mathrm{T}}$. Note that (10) can equivalently be written as

$$\dot{x}_i(t) = -\sum_{j\in\mathcal{N}_i^f} \big(x_i(t) - x_j(t)\big) + \sum_{j\in\mathcal{N}_i^f} \big(\xi_i - \xi_j\big), \quad x_i(0) = x_{i0}. \quad (11)$$

In the rest of this paper, we consider a generalized version of this formation problem that not only allows agents to achieve a desired formation but also allows them to track a dynamic, non-stationary target. In this setting, we first introduce a novel distributed control architecture using multiplex information networks to spatially alter density and orientation of the resulting formation during target tracking (Section III) and then generalize these results to allow for connectivity maintenance and collision avoidance (Section IV). Although we consider this particular formation problem in this paper, the presented multiplex information networks-based approach can be used with many other approaches to formation control.

III. Spatially Evolving Multiagent Formations

Consider a system of n agents exchanging information among each other using their local measurements according to a connected, undirected graph $\mathcal{G}$. Specifically, we propose a distributed control architecture using networks having multiple layers with the main (physical) network layer given by

$$\begin{aligned}\dot{x}_i(t) = &-\sum_{j\in\mathcal{N}_i^f} \Big(\big(x_i(t) - p_i(t) - c_i(t)\big) \\ &-\big(x_j(t) - p_j(t) - c_j(t)\big)\Big) \\ &-k_i\big(x_i(t) - p_i(t) - c_i(t)\big) + \dot{p}_i(t) + \dot{c}_i(t), \\ &x_i(0) = x_{i0}, \quad (12)\end{aligned}$$

where $x_i(t) \in \mathbb{R}^2$ denotes the state (i.e., physical position) of agent i, and $c_i(t) \triangleq [c_i^x(t), c_i^y(t)]^{\mathrm{T}} \in \mathbb{R}^2$ and

$$p_i(t) \triangleq R\big(\theta_i(t)\big) S\big(\gamma_i^x(t), \gamma_i^y(t)\big)\xi_i \in \mathbb{R}^2, \quad (13)$$

correspond to the signals locally obtained through other network layers described in the next paragraph. In (12), $k_i = 1$ only for capable (i.e., leader) agents and it is zero otherwise. Note that we implicitly assume that there exists at least one capable agent in the multiagent system. In (13), $\xi_i \in \mathbb{R}^2$ denotes the desired formation position of agent i in the sense discussed in Section II.D, $\theta_i(t) \in \mathbb{R}$ is the rotation angle of agent i that is used in its local rotation matrix given by

$$R(\theta_i(t)) \triangleq \begin{bmatrix} \cos\theta_i(t) & -\sin\theta_i(t) \\ \sin\theta_i(t) & \cos\theta_i(t) \end{bmatrix} \in \mathbb{R}^{2\times 2}, \quad (14)$$

and $\gamma_i^x(t) \in \mathbb{R}$ and $\gamma_i^y(t) \in \mathbb{R}$ are scaling factors of agent i in x and y dimensions of the planar space, respectively, that are used in its local scaling matrix given by

$$S\big(\gamma_i^x(t), \gamma_i^y(t)\big) \triangleq \mathrm{diag}\big([\gamma_i^x(t), \gamma_i^y(t)]^T\big) \in \mathbb{R}^{2\times 2}. \quad (15)$$

To define the dynamical structure of other network layers, let $\phi_i(t)$ denotes either $c_i^x(t) \in \mathbb{R}$, $c_i^y(t) \in \mathbb{R}$, $\theta_i(t) \in \mathbb{R}$, $\gamma_i^x(t) \in \mathbb{R}$, or $\gamma_i^y(t) \in \mathbb{R}$ for conciseness of the following discussion that satisfy

$$\dot{\phi}_i(t) = -q_i(t) - \tau\,\mathrm{sgn}\big(q_i(t)\big), \ \phi_i(0) = \phi_{i0}, \quad (16)$$

$$q_i(t) \triangleq \sum_{j\in\mathcal{N}_i^f} \big(\phi_i(t) - \phi_j(t)\big) + k_i\big(\phi_i(t) - \phi_0(t)\big), \quad (17)$$

where $\tau \in \mathbb{R}$ is a positive design parameter and it is assumed that $\dot{\phi}_0(t)$ is bounded. Note that in (16) and (17), $\phi_0(t)$ denotes either $c^x(t) \in \mathbb{R}$, $c^y(t) \in \mathbb{R}$, $\theta_0(t) \in \mathbb{R}$, $\gamma_0^x(t) \in \mathbb{R}$, or $\gamma_0^y(t) \in \mathbb{R}$, where $c(t) \triangleq [c^x(t), c^y(t)]^{\mathrm{T}}$ is the position of the dynamic target on a planar space, $\theta_0(t)$ is the desired rotation angle, and $\gamma_0^x(t)$ and $\gamma_0^y(t)$ are desired scaling factors, respectively. Since $k_i = 1$ only for capable agents, notice that $c(t)$, $\theta_0(t)$, $\gamma_0^x(t)$, and $\gamma_0^y(t)$ are only available to the capable agents.

Since $\dot{\phi}_0(t)$ is bounded, this implies that $|\dot{c}^x(t)| \leq \omega_{c^x}$, $|\dot{c}^y(t)| \leq \omega_{c^y}$, $|\dot{\theta}_0(t)| \leq \omega_{\theta_0}$, $|\dot{\gamma}_0^x(t)| \leq \omega_{\gamma_0^x}$, and $|\dot{\gamma}_0^y(t)| \leq \omega_{\gamma_0^y}$. In what follows, we let ω to be the largest constant among $\omega_{c^x}, \omega_{c^y}, \omega_{\theta_0}, \omega_{\gamma_0^x}$, and $\omega_{\gamma_0^y}$ without loss of generality (i.e., $|\dot{\phi}_0(t)| \leq \omega$), and set $\tau > \omega$. The next theorem shows that the multiplex information networks-based distributed controller architecture given by (12) and (16) not only allows agents to track a dynamic target but also allows them to alter density and orientation of the resulting formation.

Theorem 1. Consider the networked multiagent system given by (12) and (16), where agents exchange their local measurements using an undirected and connected graph $\mathcal{G}$. Then,

$$\lim_{t\to\infty} x_i(t) = c(t) + R(\theta_0(t)) S(\gamma_0^x(t), \gamma_0^y(t))\xi_i, \quad (18)$$

holds for all $i = 1, \ldots, n$.

Proof. We first show that $\phi_i(t)$ converges to $\phi_0(t)$ for all cases when $\phi_i(t)$ denotes either $c_i(t) \in \mathbb{R}^2$, $\theta_i(t) \in \mathbb{R}$, $\gamma_i^x(t) \in \mathbb{R}$, or $\gamma_i^y(t) \in \mathbb{R}$. For this purpose, consider the state transformation given by

$$\tilde{\phi}_i(t) \triangleq \phi_i(t) - \phi_0(t), \quad i = 1, \ldots, n. \quad (19)$$

Using (16), (17), and (19) yields

$$\dot{\tilde{\phi}}_i(t) = -q_i(t) - \tau \text{sgn}\big(q_i(t)\big) - \dot{\phi}_0(t), \quad (20)$$

$$q_i(t) = \sum_{j \in \mathcal{N}_i^f} \big(\tilde{\phi}_i(t) - \tilde{\phi}_j(t)\big) + k_i \tilde{\phi}_i(t). \quad (21)$$

By letting $\tilde{\phi}(t) \triangleq [\tilde{\phi}_1(t), \ldots, \tilde{\phi}_n(t)]^T$, (20) and (21) can be written in the compact form as

$$\dot{\tilde{\phi}}(t) = -q(t) - \tau \text{sgn}\big(q(t)\big) - \mathbf{1}_n \dot{\phi}_0(t), \quad (22)$$

$$q(t) = (\mathcal{L}(\mathcal{G}) + K)\tilde{\phi}(t), \quad (23)$$

where $K \triangleq \text{diag}([k_1, \ldots, k_n]^T)$.

Now, consider the Lyapunov function candidate $V(\tilde{\phi}) = \frac{1}{2}\tilde{\phi}^T(\mathcal{L}(\mathcal{G}) + K)\tilde{\phi}$, where its time derivative along the trajectory of (22) is given by

$$\begin{aligned} \dot{V}(\tilde{\phi}(t)) &= \tilde{\phi}^T(\mathcal{L}(\mathcal{G}) + K)\bigg(-(\mathcal{L}(\mathcal{G}) + K)\tilde{\phi}(t) \\ &\quad -\tau \text{sgn}\Big[(\mathcal{L}(\mathcal{G}) + K)\tilde{\phi}(t)\Big] - \mathbf{1}_n \dot{\phi}_0(t)\bigg) \\ &\leq -\tilde{\phi}^T(\mathcal{L}(\mathcal{G}) + K)^2\tilde{\phi}(t) \\ &\quad -\tau \|(\mathcal{L}(\mathcal{G}) + K)\tilde{\phi}(t)\|_1 \\ &\quad +|\dot{\phi}_0(t)| \|(\mathcal{L}(\mathcal{G}) + K)\tilde{\phi}(t)\|_1 \\ &\leq -\tilde{\phi}^T(\mathcal{L}(\mathcal{G}) + K)^2\tilde{\phi}(t) \\ &\quad -(\tau - \omega)\|(\mathcal{L}(\mathcal{G}) + K)\tilde{\phi}(t)\|_1. \end{aligned} \quad (24)$$

Since $\mathcal{L}(\mathcal{G}) + K \in \mathbb{S}_+^{n \times n}$ [2] and $(\tau - \omega) > 0$ by definition, $\dot{V}(\tilde{\phi}(t))$ is negative definite. Therefore, $\tilde{\phi}(t) \to 0$ as $t \to \infty$; or equivalently, $\phi_i(t) \to \phi_0(t)$ as $t \to \infty$. In other words, $p_i(t)$ will converge to $R(\theta_0(t))S(\gamma_0^x(t), \gamma_0^y(t))\xi_i$, and $c_i(t)$ will converge to $c(t)$ asymptotically.

Next, for the main network layer (12), let's consider the state transformation

$$z_i(t) \triangleq x_i(t) - p_i(t) - c_i(t), \quad i = 1, \ldots, n. \quad (25)$$

Using (25), (12) can be rewritten as

$$\dot{z}_i(t) = -\sum_{j \in \mathcal{N}_i^f} \Big(z_i(t) - z_j(t)\Big) - k_i z_i(t), \quad (26)$$

Define $z(t) \triangleq [z_1(t), \ldots, z_n(t)]^T$, then (26) can be written in the compact form as

$$\dot{z}(t) = -(\mathcal{L}(\mathcal{G}) + K) \otimes \mathrm{I}_2 z(t), \quad (27)$$

Since it is assumed that there exists at least one capable agent in the network (i.e, at least one of the diagonal elements of K is equal to 1), it follows from [Lemma 2, 23] that $\mathcal{L}(\mathcal{G}) + K \in \mathbb{S}_+^{n \times n}$, and hence, $-\big(\mathcal{L}(\mathcal{G}) + K\big)$ is a Hurwitz matrix. As a direct consequence, $z(t) \to 0$ as $t \to \infty$; or equivalently $x_i(t) \to p_i(t) + c_i(t)$. Hence, (18) holds. ■

Remark 1. The dynamical structure of other network layers given by (16) and (17) uses the sign functions in order to achieve asymptotic stability in the presence of time-varying signals $c^x(t)$, $c^y(t)$, $\theta_0(t)$, $\gamma_0^x(t)$, and $\gamma_0^y(t)$, which is consistent with the results in the networked multiagent systems literature (see, for example, [24], [25]). Note that if $c^x(t)$, $c^y(t)$, $\theta_0(t)$, $\gamma_0^x(t)$, and $\gamma_0^y(t)$ are all constants, then the results of Theorem 1 still hold without the need for the sign function in (16) and (17); that is,

$$\dot{\phi}_i(t) = -\sum_{j \in \mathcal{N}_i^f} \big(\phi_i(t) - \phi_j(t)\big) - k_i\big(\phi_i(t) - \phi_0\big). \quad (28)$$

We can also reach a similar conclusion for the case when some of these signals are constant and the respective sign functions for those are removed from (16) and (17).

Remark 2. A positive design parameter α can be used in the main network layer given by (12) as

$$\begin{aligned} \dot{x}_i(t) &= -\alpha\Big[\sum_{j \in \mathcal{N}_i^f} \Big(\big(x_i(t) - p_i(t) - c_i(t)\big) \\ &\quad -\big(x_j(t) - p_j(t) - c_j(t)\big)\Big) \\ &\quad -k_i\big(x_i(t) - p_i(t) - c_i(t)\big)\Big] + \dot{p}_i(t) + \dot{c}_i(t), \\ &\qquad x_i(0) = x_{i0}, \end{aligned} \quad (29)$$

in order to improve convergence rate of the networked multiagent system. In this case, the proof of Theorem 1 remains identical with the term $(\mathcal{L}(\mathcal{G}) + K)$ replaced with $\alpha(\mathcal{L}(\mathcal{G}) + K)$ in (27). We can also reach a similar conclusion when another positive design parameter is introduced to the other network layers given by (16) and (17).

Remark 3. The proposed algorithm can be readily generalized to a three dimensional case with $x_i(t) \in \mathbb{R}^3$. In this case, $p_i(t) \in \mathbb{R}^3$ can be redefined as

$$p_i(t) \triangleq R\big(\theta_i^x(t), \theta_i^y(t), \theta_i^z(t)\big)S\big(\gamma_i^x(t), \gamma_i^y(t), \gamma_i^z(t)\big)\xi_i, \quad (30)$$

where $\theta_i^x(t) \in \mathbb{R}$, $\theta_i^y(t) \in \mathbb{R}$, and $\theta_i^z(t) \in \mathbb{R}$ are the rotation angles corresponding to yaw, pitch, and roll, respectively, $R\big(\theta_i^x(t), \theta_i^y(t), \theta_i^z(t)\big)$ is the rotation matrix, $\gamma_i^x(t) \in \mathbb{R}$, $\gamma_i^y(t) \in \mathbb{R}$, and $\gamma_i^z(t) \in \mathbb{R}$ are the scaling factors for each dimension, and $S\big(\gamma_i^x(t), \gamma_i^y(t), \gamma_i^z(t)\big) \triangleq \text{diag}([\gamma_i^x(t), \gamma_i^y(t), \gamma_i^z(t)]^T)$ is the scaling matrix. In this case, $\phi_i(t)$ represents either $c_i^x(t)$, $c_i^y(t)$, $\theta_i^x(t)$, $\theta_i^y(t)$, $\theta_i^z(t)$, $\gamma_i^x(t)$, $\gamma_i^y(t)$, or $\gamma_i^z(t)$ that satisfies (16) and (17).

Remark 4. The proposed multiplex networks-based spatial formation control algorithm given by (12) and (16) can be also readily generalized to the case where the graph $\mathcal{G}$ is directed under the assumption that there exists at least one capable agent at the root of the spanning tree [1].

IV. Generalizations to Allow for Connectivity Maintenance and Collision Avoidance

In this section, we use tools and methods from differential potential fields (see, for example, [2], [21], [22], [26], [27] and references therein) and generalize the results of Section III to allow for connectivity maintenance and collision avoidance that are needed in real-world applications. For this purpose, we let each agent have a communication range as given in Figure 1. Specifically, we assume that two arbitrary agents can only exchange information if their relative distance is less than R, i.e., $\|x_{ij}\|_2 < R$. Furthermore, a collision region is defined as a small disk area with radius $r < d < R$ centered at agent i as depicted in this figure.

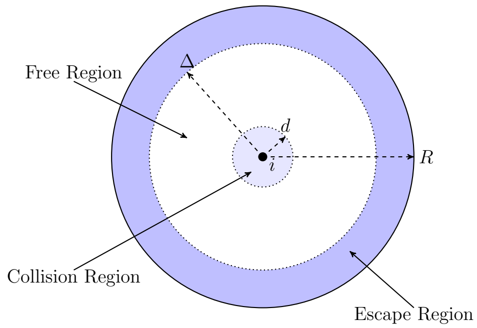


Fig. 1. Communication range of agent i.

In the same way, we define an escape region as a ring with radius $\Delta < r < R$ also centered at agent i. The region within the collision region and escape region ($d < r < \Delta$) is called free region.

We first define a (repulsive) differential potential function for the purpose of collision avoidance as

$$V_{Rij}(x_{ij}) \triangleq \begin{cases} \Big(\frac{1}{\|x_{ij}\|_2^2} - \frac{1}{d^2}\Big)^2 & \text{if } \|x_{ij}\|_2 \leq d,\, j \in \mathcal{N}_i, \\ 0 & \text{otherwise}, \end{cases} \tag{31}$$

where

$$\frac{\partial V_{Rij}(x_{ij})}{\partial x_i} = \begin{cases} -4\Big(\frac{1}{\|x_{ij}\|_2^2} - \frac{1}{d^2}\Big)\frac{x_{ij}}{\|x_{ij}\|_2^4} \\ \qquad\qquad \text{if } \|x_{ij}\|_2 \leq d,\, j \in \mathcal{N}_i, \\ 0 \qquad \text{otherwise}. \end{cases} \tag{32}$$

Now, we define a (attractive) differential potential function fur the purpose of connectivity maintenance as

$$V_{Cij}(x_{ij}) \triangleq \begin{cases} \frac{(\|x_{ij}\|_2 - \Delta)^2}{R - \|x_{ij}\|_2} & \text{if } \|x_{ij}\|_2 \geq \Delta,\, j \in \mathcal{N}_i^f, \\ 0 & \text{otherwise}, \end{cases} \tag{33}$$

where

$$\frac{\partial V_{Cij}(x_{ij})}{\partial x_i} = \begin{cases} \frac{(\|x_{ij}\|_2 - \Delta)(2R - \Delta - \|x_{ij}\|_2)}{(R - \|x_{ij}\|_2)^2 \|x_{ij}\|_2} x_{ij} \\ \qquad\qquad \text{if } \|x_{ij}\|_2 \geq \Delta,\, j \in \mathcal{N}_i^f, \\ 0 \qquad \text{otherwise}. \end{cases} \tag{34}$$

Note that $V_{Rij} = V_{Rji}$ and $V_{Cij} = V_{Cji}$ as well as $V_{Rij} = V_{Cij} = 0$ for $i = j$. The repulsive differential potential function V_{Rij} is smoothly activated when $\|x_{ij}\|_2 \leq d$ and grows to infinity as $\|x_{ij}\|_2$ approaches 0. In addition, the attractive differential potential function V_{Cij} is smoothly activated when $\|x_{ij}\|_2 \geq \Delta$ and grows to infinity as $\|x_{ij}\|_2$ approaches R. Notice that V_{Rij} applies to agent i and any agent j who are neighbor of i (i.e., $j \in \mathcal{N}_i$), while V_{Cij} only affects agent i and its formation neighbors (i.e., $j \in \mathcal{N}_i^f$). In addition, we assume that the desired distance between any two arbitrary agents lies in the free region, where this implies that the scaling factors need to be lower and upper bounded such that this assumption is not violated.

Based on the above definitions, we generalize the results of the previous section by considering the distributed spatial formation control algorithm given by

$$\begin{aligned} \dot{x}_i(t) \;=\; & -\sum_{j \in \mathcal{N}_i^f} \Big(\big(x_i(t) - p_i(t) - c_i(t)\big) \\ & -\big(x_j(t) - p_j(t) - c_j(t)\big)\Big) \\ & -k_i\big(x_i(t) - p_i(t) - c_i(t)\big) + \dot{p}_i(t) + \dot{c}_i(t), \\ & -\sum_{j \in \mathcal{N}_i} \frac{\partial V_{Rij}(x_{ij})}{\partial x_i} - \sum_{j \in \mathcal{N}_i^f} \frac{\partial V_{Cij}(x_{ij})}{\partial x_i}, \\ & x_i(0) = x_{i0}, \end{aligned} \tag{35}$$

Since we can achieve connectivity maintenance and collision avoidance by only modifying the main network layer in (12) as (35), all other network layers given by (16) remain unchanged in this setting. For the next result, we implicitly assume that agents are not stuck in local minima such that they can asymptotically converge to the free region; this assumption is standard in the networked multiagent systems literature that adopts tools and methods from differential potential fields (see Remark 5 for further discussion).

Theorem 2. Consider the networked multiagent system given by (35) and (16), where agents exchange their local measurements using an undirected and connected graph $\mathcal{G}$. If the agents are initially connected with their formation neighbors and there is no collision, then (18) holds for all $i = 1, \ldots, n$ with connectivity maintenance and collision avoidance for all $t \geq 0$.

Proof. Using the state transformation given by (25), (35) can be rewritten as

$$\begin{aligned} \dot{z}_i(t) \;=\; & -\sum_{j \in \mathcal{N}_i^f} \Big(z_i(t) - z_j(t)\Big) - k_i z_i(t) \\ & -\sum_{j \in \mathcal{N}_i} \frac{\partial V_{Rij}(x_{ij})}{\partial zi} - \sum_{j \in \mathcal{N}_i^f} \frac{\partial V_{Cij}(x_{ij})}{\partial z_i}. \end{aligned} \tag{36}$$

Note that $\frac{\partial V_{Rij}(x_{ij})}{\partial x_i(t)} = \frac{\partial V_{Rij}(x_{ij})}{\partial z_i(t)}$ and $\frac{\partial V_{Cij}(x_{ij})}{\partial x_i(t)} = \frac{\partial V_{Cij}(x_{ij})}{\partial z_i(t)}$. We define

$$V_{Ai}(z_i(t)) \triangleq \frac{1}{2} \sum_{j \in \mathcal{N}_i^f} \|(z_i(t) - z_j(t)\|_2^2 + \frac{1}{2} k_i \|z_i(t)\|_2^2, \tag{37}$$

where the partial derivative of (37) with respect to $z_i(t)$ is given by $\frac{\partial V_{Ai}(z_i(t))}{\partial z_i(t)} = \sum_{j \in \mathcal{N}_i^f} \Big(z_i(t) - z_j(t)\Big) + k_i z_i(t)$. Now, we can write

$$\begin{aligned} \dot{z}_i(t) \;=\; & -\frac{\partial V_{Ai}(z_i(t))}{\partial z_i(t)} \\ & -\sum_{j \in \mathcal{N}_i} \frac{\partial V_{Rij}(x_{ij})}{\partial z_i(t)} - \sum_{j \in \mathcal{N}_i^f} \frac{\partial V_{Cij}(x_{ij})}{\partial z_i(t)} \\ =\; & -\frac{\partial V_{Ai}(z_i(t))}{\partial z_i(t)} \\ & -\sum_{j=1}^{n} \Big(\frac{\partial V_{Rij}(x_{ij})}{\partial z_i(t)} + \frac{\partial V_{Cij}(x_{ij})}{\partial z_i(t)} \Big). \end{aligned} \tag{38}$$

Next, consider the continuously differentiable function $V : \mathcal{D}_V \times \mathbb{R}^{2n} \to \overline{\mathbb{R}}_+$ given by

$$V(\cdot) \;=\; \Big(\frac{1}{2}\sum_{i=1}^{n} V_{Ai}(z_i(t)) + \frac{1}{4}\sum_{i=1}^{n} k_i\|z_i\|_2^2\Big) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\Big(V_{Rij}(x_{ij}) + V_{Cij}(x_{ij})\Big)\Big), \quad (39)$$

where $\mathcal{D}_V = \{x \in \mathbb{R}^{2n} : \|x_{ij}\|_2 \in (0, R) \ \forall \ j \in \mathcal{N}_i^f$ and $\|x_{ij}\|_2 \in (0, \infty) \ \forall \ j \in \mathcal{N}_i \setminus \mathcal{N}_i^f\}$. For any $c > 0$, let $\Omega = \{(x, z) \in \mathcal{D}_V \times \mathbb{R}^{2n} : \ V(\cdot) \leq c\}$ denote the level sets of $V(\cdot)$ and note that

$$\begin{aligned}
\dot{V}(\cdot) &= (\nabla_z V)^T \dot{z}(t) \\
&= \sum_{i=1}^{n}\Big((\nabla_{z_i(t)} V)^T \dot{z}_i(t)\Big) \\
&= \sum_{i=1}^{n}\Bigg[\Bigg(\Big(\frac{1}{2}\nabla_{z_i(t)}\Big(\sum_{i=1}^{n} V_{Ai}(z_i(t))\Big) \\
&\quad + \frac{1}{4}\nabla_{z_i(t)}\Big(\sum_{i=1}^{n} k_i\|z_i\|_2^2\Big) \\
&\quad + \nabla_{z_i(t)}\Big(\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\Big(V_{Rij}(x_{ij}) + V_{Cij}(x_{ij})\Big)\Big)\Bigg)^T \\
&\quad \cdot \dot{z}_i(t)\Bigg] \\
&= \sum_{i=1}^{n}\Bigg[\Bigg(\nabla_{z_i(t)} V_{Ai}(z_i(t)) \\
&\quad + \sum_{j=1}^{n}\Big(\nabla_{z_i(t)} V_{Rij}(x_{ij}) + \nabla_{z_i(t)} V_{Cij}(x_{ij})\Big)\Bigg)^T \\
&\quad \cdot \dot{z}_i(t)\Bigg] \\
&= \sum_{i=1}^{n} -\|\dot{z}_i(t)\|_2^2 \leq 0, \quad t \geq 0. \qquad (40)
\end{aligned}$$

Since $\dot{V}(\cdot) \leq 0$, the level sets Ω are positively invariant, and hence, $V_{Ai}(z_i(t))$, $V_{Rij}(x_{ij})$ and $V_{Cij}(x_{ij})$ are bounded [26]. If for some $j \in \mathcal{N}_i$ such that $\|x_{ij}\|_2 \to 0$, then $V_{Rij} \to \infty$. Therefore, by the continuity of V in $\mathcal{D}_V$, it follows that $\|x_{ij}\|_2 > 0$ for all $j \in \mathcal{N}_i(t)$. Likewise, if for some $j \in \mathcal{N}_i^f$ such that $\|x_{ij}\|_2 \to R$, then $V_{Cij} \to \infty$. Once again, by the continuity of V in $\mathcal{D}_V$, it follows that $\|x_{ij}\|_2 < R$ for all $j \in \mathcal{N}_i^f$. Thus, if the agents are initially connected with their formation neighbors and there is no collision, then collision avoidance between agent i and its neighbors (i.e., $j \in \mathcal{N}_i$) and connectivity maintenance between agent i and its formation neighbors (i.e., $j \in \mathcal{N}_i^f$) are guaranteed for all $t \geq 0$.

The level sets Ω are closed by the continuity of V in $\mathcal{D}_V$ and they are bounded since $\dot{V}(\cdot) \leq 0$, and hence, they are compact. By LaSalle's invariance principle, all trajectories starting in Ω converge to the largest invariant set in $\{(x, z) \in \mathcal{D}_V \times \mathbb{R}^{2n} : \ \dot{V}(\cdot) = 0\} = \{z \in \mathbb{R}^{2n} : \dot{z}(t) = 0\}$. Based on the assumption that agents are not stuck in local minima such that they can asymptotically converge to the free region, it now follows from Theorem 1 that (18) holds. ■

Remark 5. Without the assumption that agents are not stuck in local minima, one of the following two cases occurs based on the discussion given in the last paragraph of Theorem 2:

i) Agents can converge to the free region and (18) holds.
ii) It follows from LaSalle's invariance principle and (38) that $\frac{\partial V_{Ai}(z_i(t))}{\partial z_i(t)} = -\sum_{j=1}^{n}\Big(\frac{\partial V_{Ri}(x_{ij})}{\partial z_i(t)} + \frac{\partial V_{Ci}(x_{ij})}{\partial z_i(t)}\Big)$ holds, where both left and right hand sides of this equation are not equal to zero.

Note that case ***ii)*** implies that agents are stuck in local minima. Although there are several methods to avoid local minima (see, for example, [28]–[30]), it is an open problem in the networked multiagent systems literature that adopts tools and methods from differential potential fields. Yet, for example, one can use the idea stated in [28], which assumes that agents that are stuck can be detected (e.g., agents that are not moving for a specific amount of time) and a force

$$F_{vi} \triangleq \begin{cases} F_i & \text{if } \dot{z}_i(t) = 0 \text{ and } \frac{\partial V_{Ai}(z_i(t))}{\partial z_i(t)} \neq 0, \\ 0 & \text{otherwise,} \end{cases} \qquad (41)$$

is generated to push such agents out of the local minima with F_i being a random finite value for each agent. This force can eventually yield all agents to converge to the free region such that (18) follows.

V. Illustrative Numerical Example

In this section, we present a numerical example to illustrate the results of this paper. For this purpose, consider a group of 5 agents with agent 1 being the capable agent and assume that all agents are subject to random initial conditions. We choose ξ_i for each agent to obtain the desired formation depicted in Figure 2. Specifically, to illustrate the results of Theorem 1, we use (29) with $\alpha = 5$. In addition, for (16), we use $c^x(t) = t$; $c^y(t) = \sin(t)$; $\theta_0 = 0$; and low-pass filtered version of $\psi(t) = 0.5$ for $t \in [0, 10)$, $\psi(t) = -0.25t + 3$ for $t \in [10, 11)$, $\psi(t) = 0.25$ for $t \in [11, 20)$, $\psi(t) = 0.75t - 14.75$ for $t \in [20, 21)$, and $\psi(t) = 1$ for $t \in [21, \infty)$ for both $\gamma_0^x(t)$ and $\gamma_0^y(t)$. The time derivatives of $c_i^x(t)$, $c_i^y(t)$, $\theta_i(t)$, $\gamma_i^x(t)$, and $\gamma_i^y(t)$ are all upper bounded by 5 or a smaller constant, and hence, we set $\tau = 5$.

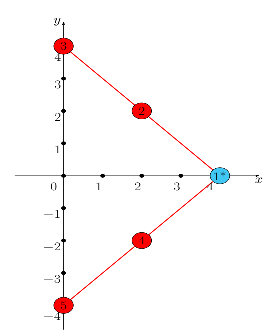


Fig. 2. Desired formation used in illustrative numerical examples.

Figure 3 shows that the considered group of agents perform target tracking while simultaneously forming, maintaining, and spatially altering their formation in time. Furthermore, Figures 4 and 5 show that $\gamma_i(t)$ converges to the desired values of the scaling factors and the state transformation variable $z_i(t)$ approaches to zero, respectively.

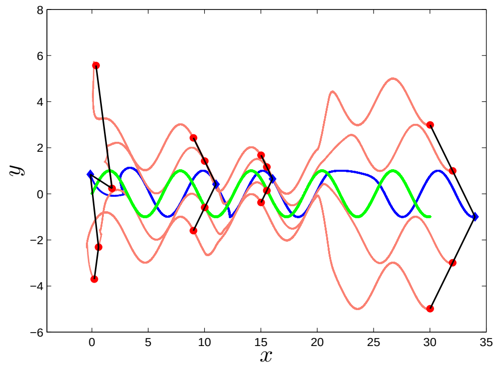
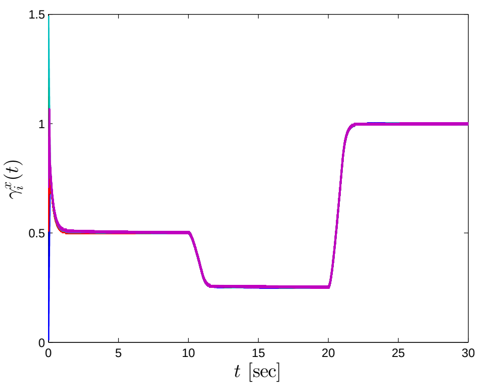
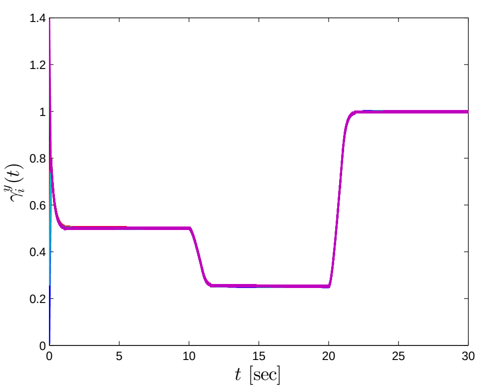


Fig. 3. Target tracking using the proposed multiplex networks-based spatial formation control algorithm in Theorem 1.



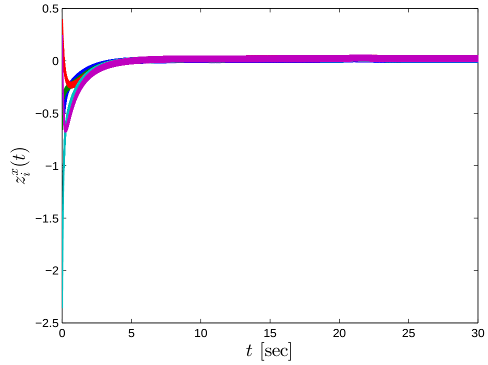
(a) Time evolution of $\gamma_i^x(t)$.



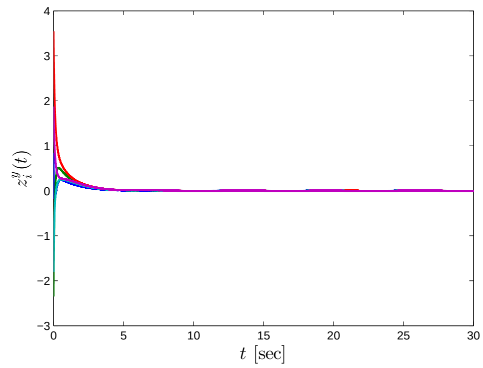
(b) Time evolution of $\gamma_i^y(t)$.

Fig. 4. Time evolution of the scaling factors in Fig. 3.

Next, we illustrate the results of Theorem 2. In particular, we add the potential field functions to (29) as in (35) and set $d = 0.5$, $\Delta = 6$, and $R = 8$, where all other design parameters remain the same. Figure 6 shows that the considered group of agents achieves the same level of



(a) Time evolution of $z_i^x(t)$.



(b) Time evolution of $z_i^y(t)$

Fig. 5. Time evolution of $z_i(t)$ in Fig. 3.

performance as in Figure 3 while maintaining connectivity and avoiding collisions. In addition, Figure 7 shows the evolution of distances between agents during $t \in [0, 5]$ seconds and illustrates collision avoidance properties of the proposed multiplex networks-based spatial formation control algorithm.

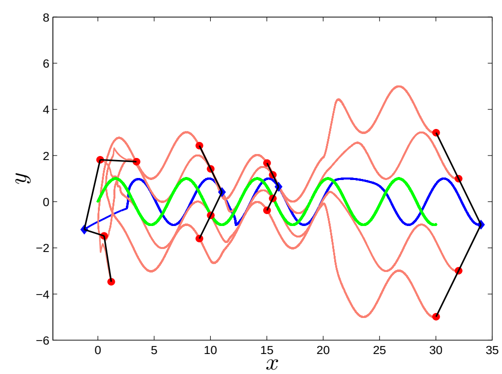


Fig. 6. Target tracking using the proposed multiplex networks-based spatial formation control algorithm in Theorem 2.

VI. Conclusion

To contribute to the previous studies in multiagent systems, we investigated how information exchange rules represented by multiplex information networks can be designed to enable spatially evolving multiagent formations. Specifically, we proposed and analyzed a distributed control architecture

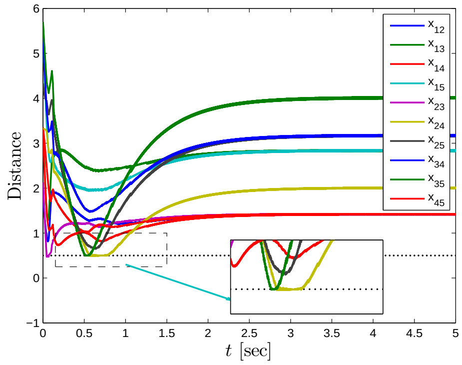


Fig. 7. Time evolution of distances between agents.

for the formation tracking problem that allows capable agents to spatially alter density and orientation of the resulting formation while tracking a dynamic, non-stationary target without requiring global information exchange ability. We further generalize these results to allow for connectivity maintenance and collision avoidance by using tools and methods from differential potential fields. Considering multi-agent operations with dramatically increasing levels of complexity, the presented multiplex networks-based approach can also be used with many other approaches in multiagent systems to enable advanced distributed information exchange rules to make these systems evolve spatially in adapting to dynamic environments and respond effectively to human interventions. Our future research will include extensions of the proposed approach to agents with high-order dynamics as well as applications of this architecture to autonomous ground and aerial vehicles.

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