

# On Control of Multiagent Formations through Local Interactions\*

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**Abstract**—We recently showed for multiagent systems with first-order agent dynamics how information exchange rules represented by a network having multiple layers (multiplex information networks) can be designed for enabling spatially evolving multiagent formations. In this paper, we generalize our earlier results for multiagent systems with general linear dynamics. Specifically, we utilize multiplex information networks for formation density control of multiagent systems. The proposed approach allows capable agents to spatially alter density of the resulting formation while tracking a target of interest — without requiring global information exchange ability, and hence, through local interactions. We provide an illustrative numerical example to demonstrate the efficacy of the proposed distributed control architecture.

## I. INTRODUCTION

Multiagent systems require advanced distributed information exchange rules for performing operations with dramatically increasing levels of complexity in order to make these systems evolve spatially for adapting dynamic environments and effectively responding to human interventions. Yet, current distributed control methods lack information exchange infrastructures to enable spatially evolving multiagent formations. This is due to the fact that these methods are designed based on information exchange rules for a network having a single layer (see, for example, [1]–[3] and references therein), which leads to multiagent formations with fixed, non-evolving spatial properties. For situations where capable agents have to control the resulting formation through these methods, they can only do so if such vehicles have global information exchange ability — that is not practical for cases involving large numbers of agents and low-bandwidth peer-to-peer communications.

In [4], [5], we showed for multiagent systems with first-order dynamics how information exchange rules represented by a network having multiple layers (multiplex information networks) can be designed for enabling spatially evolving multiagent formations. In this paper, we generalize our recent

results for multiagent systems with general linear dynamics. Specifically, we utilize multiplex information networks for formation density control of multiagent systems. The proposed approach allows capable agents to spatially alter density of the resulting formation while tracking a target of interest — without requiring global information exchange ability, and hence, through local interactions.

In particular, studies in multiplex information networks have recently emerged in the physics and networks science literatures, where they consider system-theoretic characteristics of network dynamics with multiple layers subject to intralayer and interlayer information exchange [6]–[12]. However, these studies mainly consider cases where all layers perform simple consensus algorithms and analyze the convergence of the overall multiagent systems in the presence of not only intralayer but also interlayer information exchange, and hence, they do not deal with controlling spatial properties of multiagent formations. Note that there are also recent studies on networks of networks by the authors of [13]–[15]. However, these studies deal with large-scale systems formed from smaller factor networks via graph Cartesian products, and hence, they are also not related with the contribution of this paper.

Spatial multiagent formation control and formation density control in particular is considered by the authors of [16]–[19] using approaches different from multiplex information networks. Specifically, the authors of [16]–[18] assume that some of the formation design parameters are known globally by all agents and the authors of [19] assume global knowledge of the complete network at the analysis stage. However, as previously discussed, such assumptions may not be practical in the presence of large numbers of agents and low-bandwidth peer-to-peer communications. From a data security point of view, in addition, it should be noted that one may not desire a multiagent system with all agents sharing some global information about an operation of interest. Throughout this paper, we do not make such assumptions in our multiplex information networks-based formation density control approach.

The organization of this paper is as follows. Section II introduces the necessary mathematical preliminaries to develop the main results of this paper. Section III presents

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the proposed distributed control architecture for density control of multiagent formations with general linear dynamics through local interactions. We provide an illustrative numerical example in Section IV to demonstrate the efficacy of the proposed architecture and concluding remarks are summarized in Section V.

Throughout this paper,  $\mathbb{R}$  denotes the set of real numbers,  $\mathbb{R}^n$  denotes the set of  $n \times 1$  real column vectors,  $\mathbb{R}^{n \times m}$  denotes the set of  $n \times m$  real matrices,  $\mathbb{R}_+$  denotes the set of positive real numbers,  $\mathbb{R}_+^{n \times n}$  (resp.,  $\overline{\mathbb{R}}_+^{n \times n}$ ) denotes the set of  $n \times n$  positive-definite (resp., nonnegative-definite) real matrices,  $\mathbb{S}_+^{n \times n}$  (resp.,  $\overline{\mathbb{S}}_+^{n \times n}$ ) denotes the set of  $n \times n$  symmetric positive-definite (resp., symmetric nonnegative-definite) real matrices,  $\mathbb{Z}$  denotes the set of integers,  $\mathbb{Z}_+$  (resp.,  $\overline{\mathbb{Z}}_+$ ) denotes the set of positive (resp., nonnegative) integers,  $0_n$  denotes the  $n \times 1$  vector of all zeros,  $\mathbf{1}_n$  denotes the  $n \times 1$  vector of all ones,  $0_{n \times n}$  denotes the  $n \times n$  zero matrix, and  $I_n$  denotes the  $n \times n$  identity matrix. In addition, we write  $(\cdot)^T$  for transpose,  $(\cdot)^{-1}$  for inverse,  $\|\cdot\|_2$  for the Euclidian norm,  $\|\cdot\|_F$  for the Frobenius norm,  $\lambda_{\min}(A)$  (resp.,  $\lambda_{\max}(A)$ ) for the minimum (resp., maximum) eigenvalue of the Hermitian matrix  $A$ ,  $\lambda_i(A)$  for the  $i$ -th eigenvalue of  $A$  ( $A$  is symmetric and the eigenvalues are ordered from least to greatest value), and  $\text{diag}(a)$  for the diagonal matrix with the vector  $a$  on its diagonal.

## II. MATHEMATICAL PRELIMINARIES

We first recall some basic notions from graph theory, where we refer to [2], [20] for details. In the multiagent literature, graphs are broadly adopted to encode interactions in networked systems. An undirected graph  $\mathcal{G}$  is defined by a set  $\mathcal{V}_{\mathcal{G}} = \{1, \dots, N\}$  of nodes and a set  $\mathcal{E}_{\mathcal{G}} \subset \mathcal{V}_{\mathcal{G}} \times \mathcal{V}_{\mathcal{G}}$  of edges. If  $(i, j) \in \mathcal{E}_{\mathcal{G}}$ , then the nodes  $i$  and  $j$  are neighbors and the neighboring relation is indicated with  $i \sim j$ . The degree of a node is given by the number of its neighbors. Letting  $d_i$  be the degree of node  $i$ , then the degree matrix of a graph  $\mathcal{G}$ ,  $\mathcal{D}(\mathcal{G}) \in \mathbb{R}^{N \times N}$ , is given by

$$\mathcal{D}(\mathcal{G}) \triangleq \text{diag}(d), \quad d = [d_1, \dots, d_N]^T. \quad (1)$$

A path  $i_0 i_1 \dots i_L$  is a finite sequence of nodes such that  $i_{k-1} \sim i_k$ ,  $k = 1, \dots, L$ , and a graph  $\mathcal{G}$  is connected if there is a path between any pair of distinct nodes. The adjacency matrix of a graph  $\mathcal{G}$ ,  $\mathcal{A}(\mathcal{G}) \in \mathbb{R}^{N \times N}$ , is given by

$$[\mathcal{A}(\mathcal{G})]_{ij} \triangleq \begin{cases} 1, & \text{if } (i, j) \in \mathcal{E}_{\mathcal{G}}, \\ 0, & \text{otherwise.} \end{cases}$$

The Laplacian matrix of a graph,  $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_+^{N \times N}$ , playing a central role in many graph theoretic treatments of multiagent systems, is given by

$$\mathcal{L}(\mathcal{G}) \triangleq \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G}).$$

Throughout this paper, we model a given multiagent system by a connected, undirected graph  $\mathcal{G}$ , where nodes and edges represent agents and inter-agent communication links, respectively.

Next we introduce two necessary lemmas for the results of this paper.

*Lemma 1* [2]. The spectrum of the Laplacian of a connected, undirected graph can be ordered as

$$0 = \lambda_1(\mathcal{L}(\mathcal{G})) < \lambda_2(\mathcal{L}(\mathcal{G})) \leq \dots \leq \lambda_N(\mathcal{L}(\mathcal{G})), \quad (2)$$

with  $\mathbf{1}_n$  as the eigenvector corresponding to the zero eigenvalue  $\lambda_1(\mathcal{L}(\mathcal{G}))$  and

$$\mathcal{L}(\mathcal{G})\mathbf{1}_N = 0_N. \quad (3)$$

*Lemma 2* [21]. Let

$$G = [\mu_1, \mu_2, \dots, \mu_N]^T, \quad \mu_i \in \overline{\mathbb{Z}}_+, \quad i = 1, \dots, N, \quad (4)$$

and assume at least one element of  $G$  is nonzero. Then, for the Laplacian of a connected, undirected graph,

$$F \triangleq \mathcal{L}(\mathcal{G}) + \text{diag}(G), \quad (5)$$

is a positive-definite matrix.

## III. CONTROL OF MULTIAGENT FORMATIONS WITH GENERAL LINEAR DYNAMICS

In this section, we consider a system with  $N$  agents exchanging information among each other using their local measurements according to a connected, undirected graph  $\mathcal{G}$ . Specifically, let the dynamics of each agent be described by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad x_i(0) = x_{i0}, \quad (6)$$

$$y_i(t) = Cx_i(t) \quad (7)$$

where for agent  $i$ ,  $i = 1, \dots, N$ ,  $x_i(t) \in \mathbb{R}^n$  denotes the state vector,  $u_i(t) \in \mathbb{R}^m$  denotes the control vector, and  $y_i \in \mathbb{R}^p$  denotes the output vector with  $p \leq n$ . In (6) and (7), in addition,  $A \in \mathbb{R}^{n \times n}$  denotes the system matrix,  $B \in \mathbb{R}^{n \times m}$  denotes the control input matrix, and  $C \in \mathbb{R}^{p \times n}$  denotes the output matrix such that the triple  $(A, B, C)$  is minimal.

Our objective is to design a distributed control signal  $u_i(t)$  for each agent  $i$ ,  $i = 1, \dots, N$ , such that the resulting multiagent system not only generates a desired formation but also the density of this formation is spatially altered by capable agents while tracking a target of interest — without requiring global information exchange ability, and hence, through local interactions. For this purpose, we propose the distributed control architecture given by

$$u_i(t) = -K_1 x_i(t) - K_2 z_i(t), \quad (8)$$

$$\begin{aligned} \dot{z}_i(t) = & \sum_{i \sim j} (y_i(t) - \xi_i(t) - y_j(t) + \xi_j(t)) \\ & + \mu_i (y_i(t) - \xi_i(t) - c(t)), \end{aligned} \quad (9)$$

where  $K_1 \in \mathbb{R}^{m \times n}$  and  $K_2 \in \mathbb{R}^{m \times p}$  are feedback controller gain matrices,  $z_i(t) \in \mathbb{R}^p$  is the integral state vector, and  $c(t) \in \mathbb{R}^p$  is the position of a target of interest (i.e., command to be followed by the multiagent system). In (9), in addition,  $\xi_i(t) \in \mathbb{R}^p$  denotes

$$\xi_i(t) \triangleq \gamma_i(t)\xi_i^*, \quad (10)$$

where  $\xi_i^* \in \mathbb{R}^p$  captures a desired formation objective,  $\gamma_i(t) \in \mathbb{R}$  is an additional network layer satisfying

$$\dot{\gamma}_i(t) = -\alpha \sum_{i \sim j} (\gamma_i(t) - \gamma_j(t)) - \mu_i \alpha (\gamma_i(t) - \gamma^*(t)), \quad (11)$$

with  $\gamma^*(t) \in \mathbb{R}$  being the scaling factor for the density of the resulting formation and  $\alpha > 0$ , and  $\mu_i = 1$  only for capable (i.e., leader) agents and it is zero otherwise. Throughout this paper, we assume that there is at least one capable agent in the multiagent system.

More specifically, our objective is to guarantee

$$y_i(t) \rightarrow c + \xi_i^* \gamma^*, \quad i = 1, \dots, N, \quad (12)$$

asymptotically for the case when the position of the target and the scaling factor for the density of the resulting formation are constants (i.e.,  $c(t) \equiv c$  and  $\gamma^*(t) \equiv \gamma^*$ , respectively) and approximately otherwise. To this end, we introduce two assumptions on the selection of the feedback controller gain matrices in (8).

*Assumption 1.* There exists  $K_1$  and  $K_2$  such that

$$\mathcal{H} \triangleq \begin{bmatrix} A - BK_1 & -BK_2 \\ \lambda_i C & 0 \end{bmatrix}, \quad (13)$$

is Hurwitz for all  $\lambda_i$ ,  $i = 1, \dots, n$ , where  $\lambda_i \in \text{spec}(F)$  and

$$F \triangleq \mathcal{L}(\mathcal{G}) + \text{diag}(G), \quad G \triangleq [\mu_1, \mu_2, \dots, \mu_N]^T, \quad (14)$$

with  $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_+^{N \times N}$  and  $F \in \mathbb{S}_+^{N \times N}$  by Lemmas 1 and 2, respectively.

*Assumption 2.* There exists  $K_1$  and  $K_2$  such that

$$\mathcal{J} \triangleq C\bar{A}^{-1}\bar{B}, \quad (15)$$

is invertible, where  $\mathcal{J} \in \mathbb{R}^{p \times p}$ ,

$$\bar{A} \triangleq A - BK_1 \in \mathbb{R}^{n \times n}, \quad (16)$$

and

$$\bar{B} \triangleq BK_2 \in \mathbb{R}^{n \times p}. \quad (17)$$

Next, let the aggregated vectors be given by

$$x(t) = [x_1(t), x_2(t), \dots, x_N(t)]^T \in \mathbb{R}^{Nn}, \quad (18)$$

$$\xi(t) = [\xi_1(t), \xi_2(t), \dots, \xi_N(t)]^T \in \mathbb{R}^{Np}, \quad (19)$$

and

$$\xi(t) = \begin{bmatrix} \xi_i^* & & 0 \\ & \ddots & \\ 0 & & \xi_N^* \end{bmatrix} \begin{bmatrix} \gamma_1(t) \\ \vdots \\ \gamma_N(t) \end{bmatrix} \triangleq \psi \gamma(t). \quad (20)$$

Using these vectors, (6), (7), (9), and (11) can now be written in a compact form as

$$\dot{x}(t) = (I_N \otimes A)x(t) + (I_N \otimes B)u(t), \quad (21)$$

$$y(t) = (I_N \otimes C)x(t), \quad (22)$$

$$\dot{z}(t) = (F \otimes C)x(t) - (F \otimes I_p)\psi \gamma(t) - (G \otimes I_p)c(t), \quad (23)$$

$$\dot{\gamma}(t) = -\alpha F \gamma(t) + \alpha G \gamma^*(t). \quad (24)$$

Now, consider the multiagent system given by (6), (7), where  $N$  agents exchange information among each other using their local measurements according to a connected, undirected graph  $\mathcal{G}$ . In addition, consider the distributed controller architecture given by (8), (9), and (11) subject to Assumptions 1 and 2. If the position of the target and the scaling factor for the density of the resulting formation are constants, then it can be shown that

$$\lim_{t \rightarrow \infty} y_i(t) = c + \xi_i^* \gamma^*, \quad i = 1, \dots, N. \quad (25)$$

In other words, for the case when  $c(t) \equiv c$  and  $\gamma^*(t) \equiv \gamma^*$ , the proposed distributed control architecture given by (8), (9), and (11) not only generates a desired formation but also spatially alters the density of the resulting formation.

Building on the above result, we next consider a more practical case when the position of the target and the scaling factor for the density of the resulting formation are time-varying with bounded time rates of change; that is,

$$\|\dot{c}(t)\|_2 \leq \beta_1, \quad (26)$$

$$\|\dot{\gamma}^*(t)\|_2 \leq \beta_2 \quad (27)$$

For this purpose, once again, consider the multiagent system given by (6), (7), where  $N$  agents exchange information among each other using their local measurements according to a connected, undirected graph  $\mathcal{G}$ . In addition, consider the distributed controller architecture given by (8), (9), and (11) subject to Assumptions 1 and 2. If the position of the target and the scaling factor for the density of the resulting formation are time-varying with bounded time rates of change, then it can be shown that  $y_i(t)$  converges to a neighborhood of

$$c(t) + \xi_i^* \gamma^*(t), \quad i = 1, \dots, N. \quad (28)$$

Furthermore, an ultimate bound for the distance of

$$\tilde{q}(t) = q(t) + A_q^{-1} B_q p_c(t), \quad (29)$$

can be computed as [22]

$$\|\tilde{q}(t)\|_2 \leq 2 \sqrt{\frac{\lambda_{\max}(P_q)}{\lambda_{\min}(P_q)}} \frac{\|P_q A_q^{-1} B_q\|_F (\beta_1 + \beta_2)}{\lambda_{\min}(R_q)}, \quad t \geq T. \quad (30)$$

In (30), if its right hand side is small, then the distance of (29) is small for  $t \geq T$ . It now can be shown that a small (29) implies  $y_i(t)$  to stay close to

$$c(t) + \xi_i^* \gamma^*(t), \quad i = 1, \dots, N, \quad (31)$$

for  $t \geq T$ .

Finally, in addition to controlling the density of the resulting formation, one can also control its orientation by adding an additional network layer. For example, for a two dimensional formation problem, one can use the proposed controller architecture in (8) and (9) with (10) replaced by

$$\xi_i(t) \triangleq \gamma_i(t) R(\theta_i(t)) \xi_i^* \in \mathbb{R}^2, \quad (32)$$

where

$$R(\theta_i(t)) \triangleq \begin{bmatrix} \cos \theta_i(t) & -\sin \theta_i(t) \\ \sin \theta_i(t) & \cos \theta_i(t) \end{bmatrix} \in \mathbb{R}^{2 \times 2}. \quad (33)$$

In (32),  $\gamma_i(t) \in \mathbb{R}$  and  $\theta_i(t) \in \mathbb{R}$  are additional network layers respectively satisfying (11) and

$$\dot{\theta}_i(t) = -\alpha \sum_{i \sim j} (\theta_i(t) - \theta_j(t)) - \mu_i \alpha (\theta_i(t) - \theta^*(t)), \quad (34)$$

with  $\theta^*(t) \in \mathbb{R}$  controlling the orientation of the resulting formation.

#### IV. ILLUSTRATIVE NUMERICAL EXAMPLE

In this section, we present an illustrative numerical example to demonstrate the efficacy of the proposed distributed control architecture in Section III. Specifically, consider a multiagent system with six agents exchanging information among each other using their local measurements according to a connected, undirected graph depicted in Figure 1, where the first agent is a capable agent (i.e.,  $\mu_1 = 1$  and  $\mu_i = 0$ ,  $i = 2, \dots, 6$ ). The dynamics of each agent is represented by (6) and (7) with

$$A = \begin{bmatrix} 0_{3 \times 3} & I_3 \\ A_1 & A_2 \end{bmatrix}, \quad B = \begin{bmatrix} 0_{3 \times 3} \\ I_3 \end{bmatrix}, \quad C = [I_3 \quad 0_{3 \times 3}], \quad (35)$$

where

$$A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3\omega_0^2 & 0 \\ 0 & 0 & -\omega_0^2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 2\omega_0 & 0 \\ -2\omega_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (36)$$

and

$$\omega_0 = 0.0015. \quad (37)$$

Note that (35) and (36) represent the linearized equations of the relative translational dynamics, which are described by the Clohessy-Wiltshire equations [23], and

$$x_i(t) = [\bar{\omega}_i^T(t), \dot{\bar{\omega}}_i^T(t)]^T, \quad (38)$$

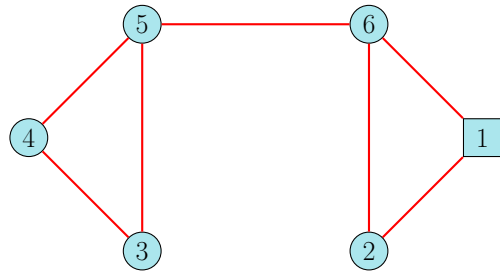


Fig. 1. A multiagent system with six agents (square denotes the capable agent, circles denote the other agents, and solid lines denote the connected, undirected graph topology.

with  $\bar{\omega}_i(t)$  being the position of agent  $i$ ,  $i = 1, \dots, 6$ , in a three dimensional space.

In this example, all agents are subject to random initial conditions and we let  $z_i(0) = 0$  and  $\gamma_i(0) = 1$ ,  $i = 1, \dots, 6$ . In addition, we choose

$$K_1 = \begin{bmatrix} 25.46 & -0.005 & 0 & 15.84 & 0 & 0 \\ 0.005 & 25.46 & 0 & 0 & 15.84 & 0 \\ 0 & 0 & 25.46 & 0 & 0 & 15.84 \end{bmatrix}, \quad (39)$$

$$K_2 = \begin{bmatrix} 14.14 & -0.003 & 0 \\ 0.003 & 14.14 & 0 \\ 0 & 0 & 14.14 \end{bmatrix}, \quad (40)$$

and

$$\alpha = 5, \quad (41)$$

where Assumptions 1 and 2 are satisfied. For  $\xi_i^*$ ,  $i = 1, \dots, 6$ , we choose  $\xi_1^* = [2 \ 0 \ 0]^T$ ,  $\xi_2^* = [1 \ -1 \ 0]^T$ ,  $\xi_3^* = [-1 \ -1 \ 0]^T$ ,  $\xi_4^* = [-2 \ 0 \ 0]^T$ ,  $\xi_5^* = [-1 \ 1 \ 0]^T$ , and  $\xi_6^* = [1 \ 1 \ 0]^T$ , which yields to an uniform hexagon desired formation on a two dimensional space. Finally, we let

$$c(t) = [0.1t \quad 2.5 \sin(0.02t) \quad 0]^T. \quad (42)$$

Figure 2A presents the results when the scaling factor for the density of the resulting formation is  $\gamma^*(t) = 0.5$  for  $t \in [0, 80)$  seconds,  $\gamma^*(t) = 1$  for  $t \in [80, 160)$  seconds, and  $\gamma^*(t) = 1.5$  for  $t \geq 160$  seconds. In addition, Figure 2B presents the results when the scaling factor for the density of the resulting formation is  $\gamma^*(t) = 1.5$  for  $t \in [0, 80)$  seconds,  $\gamma^*(t) = 1$  for  $t \in [80, 160)$  seconds, and  $\gamma^*(t) = 0.5$  for  $t \geq 160$  seconds. In both figures, we use a low-pass filter to smoothen the transition between  $\gamma^*(t)$  changes (in order to have a bounded time rate of change of  $\gamma^*(t)$ ). It is clear from these figures that the proposed distributed control architecture allows the capable agent to spatially alter density of the resulting formation while tracking a dynamic target of interest. Finally, the norm of the control signals for each agent is depicted in Figures 2C and 2D for the cases in Figures 2A and 2B, respectively.

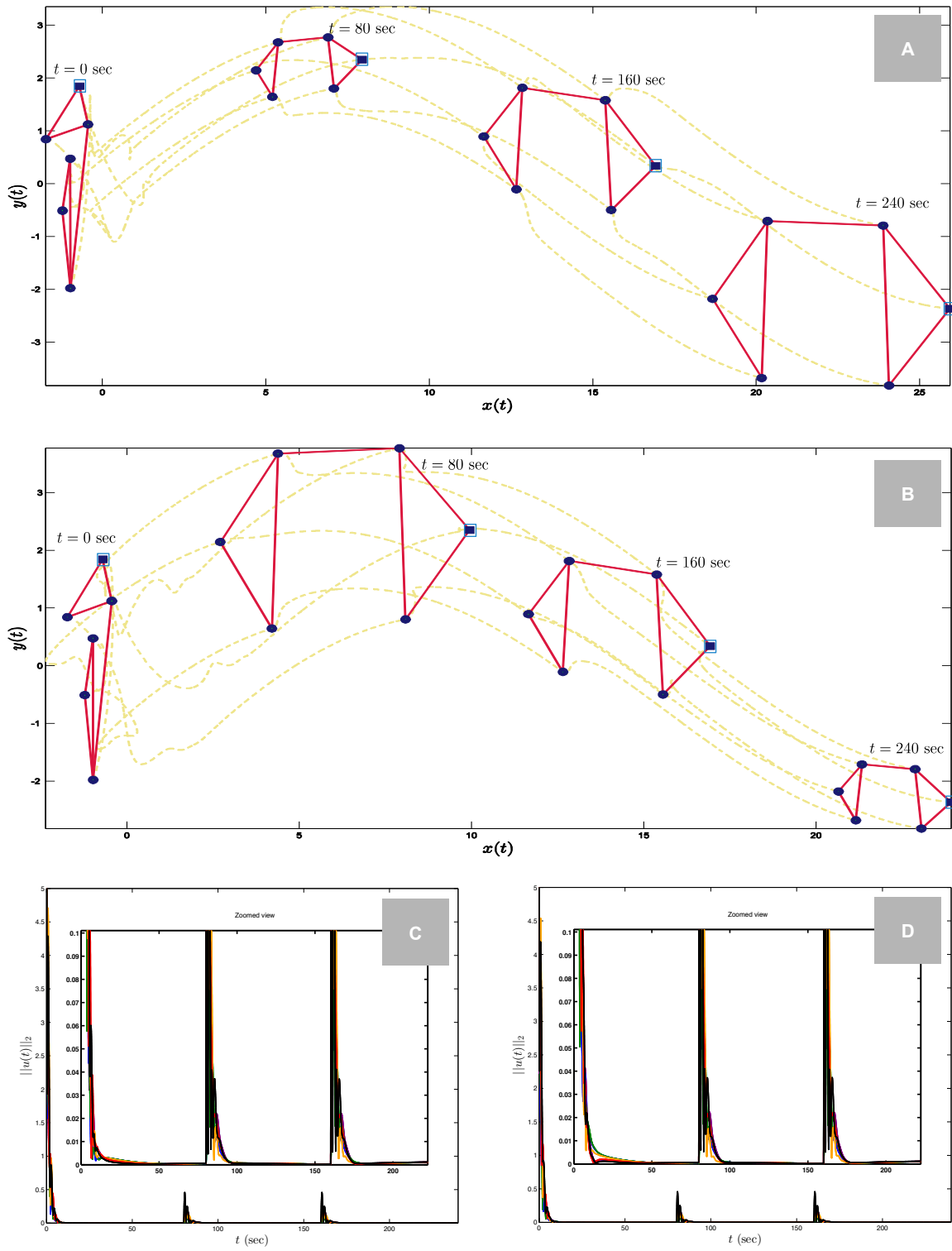


Fig. 2. Responses of the multiagent system in Figure 1 with the proposed distributed control architecture for two different scaling factors for the density of the resulting formation (A and B) and respectively the norm of the control signals of agents for each case (C and D). In A and B, square denotes the capable agent, circles denote the other agents, solid lines denote the connected, undirected graph topology, and dashed lines denote trajectories of agents on a two dimensional space.

## V. CONCLUSION

Current distributed control methods lack information exchange infrastructures to enable spatially evolving multiagent formations without having global information exchange ability. We recently showed for multiagent systems with first-order agent dynamics how information exchange rules represented by a network having multiple layers (multiplex information networks) can be designed for enabling spatially evolving multiagent formations. This paper generalized our recent results for multiagent systems with general linear dynamics. Specifically, multiplex information networks are utilized for formation density control of multiagent systems. The proposed approach allows capable agents to spatially alter density of the resulting formation while tracking a target of interest — without requiring global information exchange ability, and hence, through local interactions. An illustrative numerical result demonstrated the efficacy of the proposed methodology.

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