Resilient Control of Active-Passive Networked Multiagent Systems in the Presence of Persistent Disturbances

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An active-passive networked multiagent system consists of agents subject to inputs (active agents) and agents without any inputs (passive agents). Specifically, this class of networked multiagent systems utilizes a novel form of dynamic consensus filters, where the states of all agents converge to the average of the exogenous inputs applied only to the active agents, and it has broad practical applications including, for example, real-time situational awareness and data gathering using sensor networks and distributed control of multirobot systems. In this paper, we focus on active-passive networked multiagent systems that are subject to constant and/or harmonic exogenous disturbances. In particular, in order to improve resiliency of this class of networked multiagent systems under such persistent disturbances that can exist in adverse environments, we propose a disturbance observer-based approach and show that the proposed methodology effectively suppresses the effects of exogenous disturbances. In addition to the rigorous, system-theoretic stability and performance analysis of the proposed approach, we also show its efficacy through illustrative numerical examples.

I. Introduction

In recent years, with the rapid development of technology in computing, sensing, and communication, operations that require deploying a large number of autonomous vehicles and microsensors, have become feasible. Through communicating with each other over a network, those vehicles and/or sensors can work and coordinate as a whole system to achieve complicated tasks. These systems are known as networked multiagent systems and have widespread applications in many fields ranging from civilian operations like intelligent highways, air traffic control, geographical sampling,and military operations like battlefield environments, cooperatively transport large objects, and respond to natural disasters (see, for example, Refs. 1–5, and references therein).

To achieve global tasks, agents need to agree upon some certain quantities of interested. The average consensus in which agents converge to the average of initial values, is a well-studied class of leaderless networks (see, for example, Refs. 6–9, and references therein). However, this class of leaderless networks is not sufficient in applications to dynamic environments; for example, when each agent needs to reach

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an agreement on distance measurements between static sensors and a moving target. Motivated from this standpoint, Refs. 10–15 consider the dynamic (average) consensus problem that provides the necessary framework for a class of network applications to dynamic environments, where all agents are subject to inputs and each agent tracks the average of those inputs. Yet, it should be noted that it can be of practical importance to reach the average of the inputs only applied to a specific set of agents in the network such as a distributed network scenario when only a set of agents that are close to a target of interest can sense this target, and hence, only that set of agents are subject to inputs.

Our earlier work documented in Refs. 16 and 17 addresses this problem by introducing an active-passive networked multiagent system, which consists of agents subject to inputs (active agents) and agents without any inputs (passive agents). Specifically, this new class of networked multiagent systems utilizes a novel form of dynamic consensus filters, where the states of all agents converge to the average of the exogenous inputs applied only to the active agents, and it has broad practical applications including, for example, real-time situational awareness and data gathering using sensor networks and distributed control of multirobot systems.

In this paper, we focus on active-passive networked multiagent systems that are subject to constant and/or harmonic exogenous disturbances. In particular, in order to improve resiliency of this class of networked multiagent systems under such persistent disturbances that can exist in adverse environments,[18–20] we propose a disturbance observer-based approach and show that the proposed methodology effectively suppresses the effects of exogenous disturbances. Stability and performance of the proposed approach is rigorously analyzed using tools and methods from systems theory and Lyapunov methods. Several illustrative numerical examples are provided to demonstrate the efficacy of the proposed disturbance observer-based approach to active-passive networked multiagent systems.

The organization of this paper is as follows. In Section II, we introduce the notation and necessary lemmas for the main results of this paper. Section III overviews the active-passive networked multiagent system documented in Refs. 16 and 17. Proposed disturbance observer-based approach to this class of networked multiagent systems is given in Section IV, where this section includes stability and performance analysis of this approach. Finally, illustrative numerical examples are given in Section V and concluding remarks are summarized in Section VI.

II. Notation and Mathematical Preliminaries

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrices, $\mathbb{R}_+$ denotes the set of positive real numbers, $\mathbb{R}_+^{n\times n}$ (resp., $\overline{\mathbb{R}}_+^{n\times n}$) denotes the set of $n \times n$ positive-definite (resp., nonnegative-definite) real matrices, $\mathbb{S}_+^{n\times n}$ (resp., $\overline{\mathbb{S}}_+^{n\times n}$) denotes the set of $n \times n$ symmetric positive-definite (resp., symmetric nonnegative-definite) real matrices, 0_n denotes the $n \times 1$ vector of all zeros, $\mathbf{1}_n$ denotes the $n \times 1$ vector of all ones, $0_{n\times n}$ denotes the $n \times n$ zero matrix, and I_n denotes the $n \times n$ identity matrix. In addition, we write $(\cdot)^{\mathrm{T}}$ for transpose, $(\cdot)^{-1}$ for inverse, $(\cdot)^{\dagger}$ for generalized inverse, $\|\cdot\|_2$ for the Euclidian norm, $\|\cdot\|_F$ for the Frobenius norm, $\lambda_{\min}(A)$ (resp., $\lambda_{\max}(A)$) for the minimum (resp., maximum) eigenvalue of the Hermitian matrix A, $\lambda_i(A)$ for the i-th eigenvalue of A (A is symmetric and the eigenvalues are ordered from least to greatest value), $\mathrm{diag}(a)$ for the diagonal matrix with the vector a on its diagonal.

Next, we recall some basic notions from graph theory (see Refs. 6 and 21 for details). In the multiagent literature, graphs are broadly adopted to encode interactions in networked multiagent systems. An undirected graph $\mathcal{G}$ is defined by a set $\mathcal{V}_{\mathcal{G}} = \{1, \ldots, n\}$ of nodes and a set $\mathcal{E}_{\mathcal{G}} \subset \mathcal{V}_{\mathcal{G}} \times \mathcal{V}_{\mathcal{G}}$ of edges. If $(i, j) \in \mathcal{E}_{\mathcal{G}}$, then the nodes i and j are neighbors and the neighboring relation is indicated with $i \sim j$. The degree of a node is given by the number of its neighbors. Letting d_i be the degree of node i, then the degree matrix of a graph $\mathcal{G}$, $\mathcal{D}(\mathcal{G}) \in \mathbb{R}^{n\times n}$, is given by $\mathcal{D}(\mathcal{G}) \triangleq \mathrm{diag}(d)$, $d = [d_1, \ldots, d_n]^{\mathrm{T}}$. A path $i_0 i_1 \ldots i_L$ is a finite sequence of nodes such that $i_{k-1} \sim i_k$, $k = 1, \ldots, L$, and a graph $\mathcal{G}$ is connected if there is a path between any pair of distinct nodes. The adjacency matrix of a graph $\mathcal{G}$, $\mathcal{A}(\mathcal{G}) \in \mathbb{R}^{n\times n}$, is given by

$$[\mathcal{A}(\mathcal{G})]_{ij} \triangleq \begin{cases} 1, & \text{if } (i, j) \in \mathcal{E}_{\mathcal{G}}, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

The Laplacian matrix of a graph, $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_+^{n\times n}$, playing a central role in many graph theoretic treatments of multiagent systems, is given by

$$\mathcal{L}(\mathcal{G}) \triangleq \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G}). \tag{2}$$

The spectrum of the Laplacian of a connected, undirected graph can be ordered as

$$0 = \lambda_1(\mathcal{L}(\mathcal{G})) < \lambda_2(\mathcal{L}(\mathcal{G})) \le \cdots \le \lambda_n(\mathcal{L}(\mathcal{G})), \tag{3}$$

with $\mathbf{1}_n$ as the eigenvector corresponding to the zero eigenvalue $\lambda_1(\mathcal{L}(\mathcal{G}))$ and $\mathcal{L}(\mathcal{G})\mathbf{1}_n = 0_n$ and $\mathrm{e}^{\mathcal{L}(\mathcal{G})}\mathbf{1}_n = \mathbf{1}_n$.

Throughout this paper, we model a given multiagent system by a connected, undirected graph $\mathcal{G}$, where nodes and edges represent agents and inter-agent communication links, respectively.

Lemma 1[16] . Let $K = \mathrm{diag}(k)$, $k = [k_1, k_2, \ldots, k_n]^{\mathrm{T}}$, $k_i \in \overline{\mathbb{R}}_+$, $i = 1, \ldots, n$, and assume that at least one element of k is nonzero. Then, for the Laplacian of a connected, undirected graph,

$$\mathcal{F}(\mathcal{G}) \triangleq \mathcal{L}(\mathcal{G}) + K \in \mathbb{S}_+^{n\times n}, \tag{4}$$

and $\det(\mathcal{F}(\mathcal{G})) \neq 0$.

Lemma 2[22] . The Laplacian of a connected, undirected graph satisfies $\mathcal{L}(\mathcal{G})\mathcal{L}^{\dagger}(\mathcal{G}) = \mathrm{I}_n - \frac{1}{n}\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}$.

III. Overview of Active–Passive Networked Multiagent Systems

In this section, we briefly overview the active-passive networked multiagent systems approach introduced in 16 and 17. Specifically, we consider a system of n agents exchanging information among each other using their local measurements according to a connected, undirected graph $\mathcal{G}$. In addition, consider that there exists $m \ge 1$ exogenous inputs that interact with this system.

Definition 1. If agent i, $i = 1, \ldots, n$, is subject to one or more exogenous inputs (resp., no exogenous inputs), then it is an active agent (resp., passive agent).

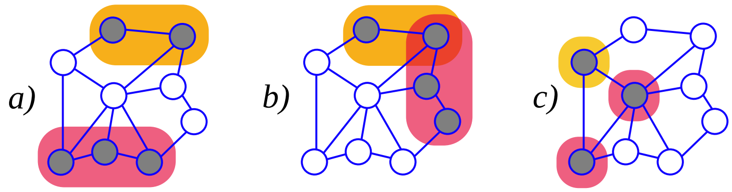

Figure 1. An active-passive networked multiagent system with a) two non-overlapping non-isolated inputs, b) two overlapping non-isolated inputs, and c) two non-overlapping inputs, where one of them is isolated and the other one is non-isolated (lines denote communication links, gray circles denote active agents, white circles denote passive agents, and shaded areas denote the exogenous inputs interacting with this system).[17]

Definition 2. If an exogenous input interacts with only one agent (resp., multiple agents), then it is an isolated input (resp., non-isolated input) (see Figure 1 adopted from Ref. 17).

The approach presented in Refs. 16 and 17 deals with the problem of driving the states of all (active and passive) agents to the average of the applied exogenous inputs. For this purpose, the following integral action-based distributed control algorithm is proposed

$$\dot{x}_i(t) = -\alpha \sum_{i\sim j} \big(x_i(t) - x_j(t)\big) + \sum_{i\sim j} \big(\xi_i(t) - \xi_j(t)\big) - \alpha \sum_{i\sim h} (x_i(t) - c_h(t)), \quad x_i(0) = x_{i0}, \tag{5}$$

$$\dot{\xi}_i(t) = -\gamma \Big[\sum_{i\sim j} \big(x_i(t) - x_j(t)\big) + \sigma \xi_i(t) \Big], \quad \xi_i(0) = \xi_{i0}, \tag{6}$$

where $x_i(t) \in \mathbb{R}$ and $\xi_i(t) \in \mathbb{R}$ denote the state and the integral action of agent i, $i = 1, \ldots, n$, respectively, $c_h(t) \in \mathbb{R}$, $h = 1, \ldots, m$, denotes an exogenous input sensed by this agent, $\alpha \in \mathbb{R}_+$, and $\gamma \in \mathbb{R}_+$. Note that $i \sim h$ notation indicates the exogenous inputs that an agent is subject to, which is similar to the $i \sim j$ notation indicating the neighboring relation between agents.

Remark 1. The results of Refs. 16 and 17 show that the states of all agents converge to (resp., converge to an adjustable neighborhood of) the average of the constant (resp., time-varying) inputs applied to the active agents under the assumption that no disturbances act on the state of any agents. For further insights regarding (5) and (6), see Examples 1, 2, 3, 4, and 5 of Ref. 17. In addition, we refer to Section VI of Ref. 17 that compare (5) and (6) with other existing approaches to networked multiagent systems.

IV. Resilient Control of Active-Passive Networked Multiagent Systems using Disturbance Observer

In this section, we introduce a disturbance observer-based approach for active-passive networked multi-agent systems to achieve resilience against constant and harmonic exogenous disturbances.

A. Problem Formulation

Consider the proposed resilient active–passive networked multiagent system algorithm subjected to exogenous disturbances given by

$$\dot{x}_i(t) = -\alpha \sum_{i\sim j} \big(x_i(t) - x_j(t)\big) + \gamma \sum_{i\sim j} \big(\xi_i(t) - \xi_j(t)\big) - \alpha \sum_{i\sim h} (x_i(t) - c_h(t)) + d_i(t) - \hat{d}_i(t), \; x_i(0) = x_{i0}, \quad (7)$$

$$\dot{\xi}_i(t) = -\gamma\Big[\sum_{i\sim j} \big(x_i(t) - x_j(t)\big) + \sigma\xi_i(t)\Big], \; \xi_i(0) = \xi_{i0}, \quad (8)$$

where $x_i(t) \in \mathbb{R}$ and $\xi_i(t) \in \mathbb{R}$ denote the state and the integral action of agent i, $i = 1, \ldots, n$, respectively, $d_i(t) \in \mathbb{R}$ is the exogenous constant and/or harmonic disturbances affecting the algorithm, $\hat{d}_i(t) \in \mathbb{R}$ is the disturbance estimate to be defined, and $c_h(t) \in \mathbb{R}$, $h = 1, \ldots, m$, denotes inputs sensed by active agents, where $\alpha \in \mathbb{R}_+$, and $\gamma \in \mathbb{R}_+$. It is reasonable to assume that the applied input $c_h(t)$ and its time derivative $\dot{c}_h(t)$ are bounded. Note that $i \sim h$ notation indicates the exogenous inputs that an agent is subject to, which is similar to the $i \sim j$ notation indicating the neighboring relation between agents.

To suppress the effects of exogenous constant and/or harmonic disturbances for closely synchronizing agent outputs, we now make the following assumptions.

Assumption 1. The exogenous disturbances are generated by linear exogenous systems[23]

$$\begin{cases} \dot{w}_i(t) = A_i w_i(t), \quad w_i(0) = w_{i0}, & (9) \\ d_i(t) = C_i w_i(t), & (10) \end{cases}$$

where $w_i(t) \in \mathbb{R}^p$ is the internal state of exogenous disturbance affecting agent i, and $A_i \in \mathbb{R}^{p\times p}$ and $C_i \in \mathbb{R}^{1\times p}$ are coefficient matrices. In addition, the system given by (9) and (10) is considered neutral stable, which implies that the disturbance is persistent.[23]

Assumption 2. The dynamics given by (9) and (10) is observable.

Throughout this paper, A_i and C_i are treated as known matrices based on some knowledge about the disturbance on the networked multiagent system. Next, for brevity, we define

$$f_i(t) \triangleq -\alpha \sum_{i\sim j} \big(x_i(t) - x_j(t)\big) + \gamma \sum_{i\sim j} \big(\xi_i(t) - \xi_j(t)\big) - \alpha \sum_{i\sim h} (x_i(t) - c_h(t)), \quad (11)$$

to be the disturbance free system dynamics, and hence, (7) can be equivalently rewritten as

$$\dot{x}_i(t) = f_i(t) + d_i(t) - \hat{d}_i(t). \quad (12)$$

Now, utilizing the results in Ref. 23, we propose the agent-wise disturbance observers for the active–

passive networked multiagent system given by (7) and (8) as

$$\begin{cases} \dot{z}_i(t) = (A_i - K_iC_i)\big(z_i(t) + K_ix_i(t)\big) - K_i\big(f_i(t) - \hat{d}_i(t)\big), \quad z_i(0) = z_{i0} & (13) \\ \hat{w}_i(t) = z_i(t) + K_ix_i(t), & (14) \\ \hat{d}_i(t) = C_i\hat{w}_i(t), & (15) \end{cases}$$

where $z_i(t) \in \mathbb{R}^p$ is internal state variables of the agent-wise disturbance observers, $K_i \in \mathbb{R}^{p\times 1}$ is the observer gain such that $A_i - K_iC_i$ is Hurwitz, and $\hat{w}_i(t) \in \mathbb{R}^p$ is the agent-wise estimate of $w_i(t)$.

This concludes the setup of our problem. In the next section, we present the stability and performance guarantees for the system given by (7), (8), (13), (14), and (15).

B. Stability and Performance Analysis

In this section, we analyze the stability and performance of the active–passive networked multiagent system given by (7),(8), (13), (14), and (15). Specially, we first present and analyze the disturbance estimator error dynamics. We then demonstrate the stability of the disturbed active–passive networked multiagent system under the disturbance estimator given by (13), (14), and (15). Finally, we characterize ultimate performance bound for the overall closed-loop system.

To begin with, consider the disturbance estimator error given by

$$\mu_i(t) \triangleq w_i(t) - \hat{w}_i(t). \tag{16}$$

The time derivative of (16) can be given by

$$\begin{aligned} \dot{\mu}_i(t) &= \dot{w}_i(t) - \dot{\hat{w}}_i(t) \\ &= \dot{w}_i(t) - \dot{z}_i(t) - K_i\dot{x}_i(t) \\ &= \dot{w}_i(t) - (A_i - K_iC_i)\big(z_i(t) + K_ix_i(t)\big) + K_i\big(f_i(t) - \hat{d}_i(t)\big) - K_i\big(f_i(t) + d_i(t) - \hat{d}_i(t)\big) \\ &= A_iw_i(t) - (A_i - K_iC_i)\hat{w}_i(t) - K_id_i(t) \\ &= (A_i - K_iC_i)\mu_i(t), \quad \mu_i(0) = \mu_{i0}. \end{aligned} \tag{17}$$

Note that, $d_i(t) - \hat{d}_i(t) = C_i\big(\omega_i(t) - \hat{\omega}_i(t)\big) = C_i\mu_i(t)$. Note also that $\lim_{t\to\infty}\big(d_i(t) - \hat{d}_i(t)\big) = 0$, when $\lim_{t\to\infty}\mu_i(t) = 0$.

Now, we analyze the active–passive networked multiagent system dynamics given by (7) and (8). For this purpose, let

$$x(t) \triangleq \big[x_1(t), x_2(t), \ldots, x_n(t)\big]^{\mathrm{T}} \in \mathbb{R}^n, \tag{18}$$

$$\xi(t) \triangleq \big[\xi_1(t), \xi_2(t), \ldots, \xi_n(t)\big]^{\mathrm{T}} \in \mathbb{R}^n, \tag{19}$$

$$c(t) \triangleq \big[c_1(t), c_2(t), \ldots, c_m(t), 0, \ldots, 0\big]^{\mathrm{T}} \in \mathbb{R}^n, \tag{20}$$

where we assume $m \le n$ for the ease of notation without loss of generality.[17] We can write (7) and (8) in the compact form as

$$\dot{x}(t) = -\alpha\mathcal{F}(\mathcal{G})x(t) + \gamma\mathcal{L}(\mathcal{G})\xi(t) + \alpha K_2 c(t) + (I_n \otimes C)\mu(t), \quad x(0) = x_0, \tag{21}$$

$$\dot{\xi}(t) = -\gamma\mathcal{L}(\mathcal{G})x(t) - \gamma\sigma\xi(t), \qquad \xi(0) = \xi_0, \tag{22}$$

where $\mu(t) = \left[\mu_1^{\mathrm{T}}(t), \mu_2^{\mathrm{T}}(t), \ldots, \mu_n^{\mathrm{T}}(t)\right]^{\mathrm{T}} \in \mathbb{R}^{np}$, $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_+^{n\times n}$ is the Laplacian matrix, and $\mathcal{F}(\mathcal{G}) \triangleq \mathcal{L}(\mathcal{G}) + K_1 \in \mathbb{S}_+^{n\times n}$ as a consequence of Lemma 1 with

$$K_1 \triangleq \mathrm{diag}([k_{1,1}, k_{1,2}, \ldots, k_{1,n}]^{\mathrm{T}}) \in \overline{\mathbb{S}}_+^{n\times n}, \tag{23}$$

with $k_{1,i} \in \overline{\mathbb{Z}}_+$ denoting the number of the exogenous inputs applied to agent i, $i = 1, \ldots, n$, and

$$K_2 \triangleq \begin{bmatrix} k_{2,11} & k_{2,12} & \cdots & k_{2,1n} \\ k_{2,21} & k_{2,22} & \cdots & k_{2,2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{2,n1} & k_{2,n2} & \cdots & k_{2,nn} \end{bmatrix} \in \mathbb{R}^{n\times n}, \tag{24}$$

with $k_{2,ih} = 1$ if the exogenous input $c_h(t)$, $h = 1, \ldots, m$, is applied to agent i, $i = 1, \ldots, n$, and $k_{2,ih} = 0$ otherwise. In addition, note that $k_{1,i} = \sum_{j=1}^{n} k_{2,ij}$. We refer the reader to Refs. 16 and 17 for detailed examples illustrating the construction of K_1 and K_2 matrices.

Next, we define

$$\delta(t) \triangleq x(t) - \epsilon(t)\mathbf{1}_n \in \mathbb{R}^n, \tag{25}$$

$$\epsilon(t) \triangleq \frac{\mathbf{1}_n^{\mathrm{T}} K_2 c(t)}{\mathbf{1}_n^{\mathrm{T}} K_2 \mathbf{1}_n} \in \mathbb{R}, \tag{26}$$

where $\delta(t)$ is the error between $x_i(t), i = 1, \ldots, n$, and the average of the applied inputs $\epsilon(t)$.

Using (21), (22), in (25), the time derivative of the state error can be given by

$$\dot{\delta}(t) = -\alpha\mathcal{F}(\mathcal{G})\delta(t) + \gamma\mathcal{L}(\mathcal{G})\xi(t) - \alpha L_c K_2 c(t) + (I_n \otimes C)\mu(t) + p_1(t), \tag{27}$$

where $p_1(t) \triangleq -\dot{\epsilon}\mathbf{1}_n$ and

$$L_c \triangleq \frac{K_1 \mathbf{1}_n \mathbf{1}_n^{\mathrm{T}}}{\mathbf{1}_n^{\mathrm{T}} K_2 \mathbf{1}_n} - \mathrm{I}_n. \tag{28}$$

In addition, consider

$$e(t) \triangleq \xi(t) - \frac{\alpha}{\gamma}\mathcal{L}^{\dagger}(\mathcal{G}) L_c K_2 c(t) \in \mathbb{R}^n, \tag{29}$$

and note that $\mathbf{1}_n^{\mathrm{T}} L_c(t) = 0$. The closed loop system error dynamics can now be given by

$$\dot{\delta}(t) = -\alpha\mathcal{F}(\mathcal{G})\delta(t) + \gamma\mathcal{L}(\mathcal{G})e(t) + (I_n \otimes C)\mu(t) + p_1(t), \tag{30}$$

$$\dot{e}(t) = -\gamma\mathcal{L}\delta(t) - \gamma\sigma e(t) + p_2(t), \tag{31}$$

where $p_2(t) \triangleq -\alpha\mathcal{L}^{\dagger} L_c K_2\big(\sigma c(t) + \frac{1}{\gamma}\dot{c}(t)\big)$. Since $c(t)$ and $\dot{c}(t)$ are bouded by definition, one can write

$$\|p_1(t)\|_2 \leq p_1^* \triangleq n\dot{\epsilon}^*, \tag{32}$$

$$\|p_2(t)\|_2 \leq p_2^* \triangleq \alpha\|\mathcal{L}^{\dagger} L_c K_2\|_F \bar{c}^*, \tag{33}$$

with $\|\dot{\epsilon}(t)\|_2 \leq \dot{\epsilon}^{*2}$ and $\|\sigma c(t) + \frac{1}{\gamma}\dot{c}(t)\|_2 \leq \bar{c}^*$.

We are now ready to state the main result of this paper.

Theorem 1. Consider the active–passive network multiagent system given by (7) and (8), where agents are subjected to disturbance and exchange information using local measurements according to a connected, undirected graph $\mathcal{G}$. Under the disturbance observer architecture given by (13), (14), and (15) subject to Assumptions 1 and 2, the closed-loop error dynamics defined by (30) and (31) are uniformly bounded.

Proof. We begin by noting that the disturbance estimator error dynamics given by (17) can be written in the compact form as

$$\dot{\mu}(t) = \bar{A}\mu(t), \qquad \bar{A} \triangleq \begin{bmatrix} A_1 - K_1C_1 & 0 & \cdots & 0 \\ 0 & A_2 - K_2C_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_n - K_nC_n \end{bmatrix} \tag{34}$$

Note that, since each $(A_i - K_iC_i)$ is Hurwitz, then so is $\bar{A}$. Therefore, the trajectory of $\mu(t)$ satisfies $\mu(t) = e^{\bar{A}t}\mu(0)$. From this, note also that $\lim_{t\to\infty} \mu(t) = 0$ and $\|\mu(t)\|_2 \leq \mu^* \triangleq \|\mu(0)\|_2$.

Next, consider the Lyapunov function candidate given by

$$V(\delta, e) = \frac{1}{2}\delta^{\mathrm{T}}\delta + \frac{1}{2}e^{\mathrm{T}}e, \tag{35}$$

Note that, $V(0,0) = 0$ and $V(\delta, e) > 0$ for all $(\delta, e) \neq 0$. Taking the time derivative of (35) along the trajectories of (30) and (31), we have

$$\begin{aligned} \dot{V}(\cdot) &= -\alpha\delta^{\mathrm{T}}(t)\mathcal{F}(\mathcal{G})\delta(t) + \delta^{\mathrm{T}}(t)(\mathrm{I}_n \otimes C)\mu(t) + \delta^{\mathrm{T}}(t)p_1(t) - \gamma\sigma e^{\mathrm{T}}(t)e(t) + e^{\mathrm{T}}(t)p_2(t) \\ &\leq -\alpha\lambda_{min}\big(\mathcal{F}(\mathcal{G})\big)\|\delta(t)\|_2^2 + \|\delta(t)\|_2\|\mathrm{I}_n \otimes C\|_F\mu^* + \|\delta(t)\|_2 p_1^* - \gamma\sigma\|e(t)\|_2^2 + \|e(t)\|_2 p_2^* \\ &\leq -\alpha\lambda_{min}\big(\mathcal{F}(\mathcal{G})\big)\|\delta(t)\|_2\big(\|\delta(t)\|_2 - \phi_1\big) - \gamma\sigma\|e(t)\|_2\big(\|e(t)\|_2 - \phi_2\big) \end{aligned} \tag{36}$$

where

$$\phi_1 \triangleq \Big(\|\mathrm{I}_n \otimes C\|_F \mu^* + p_1^*\Big)/\alpha\lambda_{min}\big(\mathcal{F}(\mathcal{G})\big), \tag{37}$$

$$\phi_2 \triangleq p_2^*/(\gamma\sigma). \tag{38}$$

Note that $\dot{V}(\cdot) \leq 0$ when $\|\delta\|_2 \geq \phi_1$ and $\|e(t)\|_2 \geq \phi_2$, and hence the closed-loop error dynamics given by (30) and (31) are uniformly bounded. ■

The next result is now immediate that characterizes ultimate performance bound for the error between $x_i(t),\ i = 1, \ldots, n$, and the average of the apply inputs $\epsilon(t)$.

Corollary 1. Consider the active–passive network multiagent system given by (7) and (8), where agents are subjected to disturbance and exchange information using local measurements according to a connected, undirected graph $\mathcal{G}$. Then the ultimate performance bound of $\delta(t)$ for $t \geq T$ is given by

$$\|\delta(t)\|_2^2 \leq \frac{\Big(\|\mathrm{I}_n \otimes C\|_F \mu^* + n\dot{\epsilon}^*\Big)^2}{\alpha^2 \lambda_{min}^2\big(\mathcal{F}(\mathcal{G})\big)} + \frac{\alpha^2}{\sigma^2} \frac{(\|\mathcal{L}^\dagger L_c K_2\|_F \bar{c}^*)^2}{\gamma^2} \tag{39}$$

Proof. From the proof of Theorem 1, $V(\cdot) \leq 0$ outside the compact set given by

$$\mathcal{S} \triangleq \big\{(\delta(t), e(t)) : \|\delta(t)\|_2 \leq \phi_1\big\} \cap \big\{(\delta(t), e(t)) : \|e(t)\|_2 \leq \phi_2\big\}. \tag{40}$$

Therefore, the evolution of $V(\delta(t), e(t))$ is upper bounded by

$$V(\delta(t), e(t)) \leq \max_{(\delta(t), e(t)) \in \mathcal{S}} V(\delta(t), e(t)) = \frac{1}{2}\big(\phi_1^2 + \phi_2^2\big), \quad t \geq T \tag{41}$$

Using $\frac{1}{2}\delta(t)^{\mathrm{T}}\delta(t) \leq V(\delta(t), e(t))$ in (41), then (39) is immediate. ■

Remark 2. Corollary 1 implies that if we judiciously choose α, γ and σ such that $\frac{1}{\alpha^2}$ and $\frac{\alpha^2}{\sigma^2\gamma^2}$ are small, then (39) is small for $t \geq T$. Hence, the distance between $x_i(t),\ i = 1, \ldots, n$, and the average of the applied inputs $\epsilon(t)$ can be made arbitrarily small in the presence of exogenous disturbances.

Remark 3. If the input $c(t)$ is a vector of constants, the bounds in (37) and (38) become

$$\phi_1 \triangleq \|(I_n \otimes C)\|_F \mu^*/\Big(\alpha\lambda_{min}\big(\mathcal{F}(\mathcal{G})\big)\Big), \tag{42}$$

$$\phi_2 \triangleq \alpha\|\mathcal{L}^\dagger L_c K_2\|\|c\|_2/\gamma, \tag{43}$$

which are less conservative.

Remark 4. The disturbance observer given by (13), (14), and (15) can attenuate constant and harmonic disturbances with known frequency but unknown amplitude and phase.

V. Numerical Examples

In this section, we first demonstrate the efficacy of the proposed disturbance observer architecture in Section IV for active–passive networked multiagent system with time-varying exogenous inputs. We then consider a two-level control hierarchy of active–passive networked multiagent system in the second example.

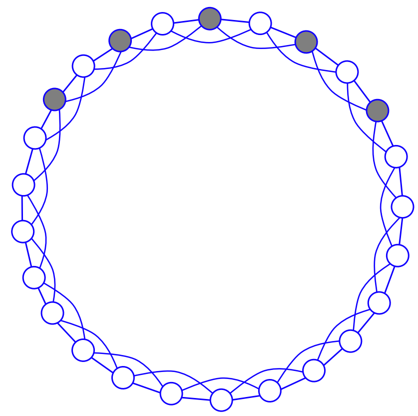

Figure 2. Communication graph of 25 agents (lines denote communication links, gray circles denote active agents, and white circles denote passive agents).

Example 1: In this example, we consider a network system with 5 active agents and 20 passive agents exchanging information under the connected, undirected graph $\mathcal{G}$ as shown in Figure 2. Active agents are subjected to isolated time-varying exogenous inputs given by $c_1(t) = 0.3075 + \sin(0.5t)$, $c_2(t) = -1.2571 + \sin(t)$, $c_3(t) = -0.8655 + \sin(1.5t)$, $c_4(t) = -0.1765 + \sin(2t)$ and $c_5(t) = 0.7914 + \sin(2.5t)$. Let all agents have arbitrary initial conditions and $\xi_i(0) = 0, i = 1, \ldots, n$. A disturbance in the form $d_i = b_i \sin(2t + p_i)$ is injected to each agent where $b_i \in [-26, 40]$ and $p_i \in [-3.5, 4.5]$ are generated randomly for $i = 1, \ldots, n$. Figure 3 shows the response of the networked multiagent system given by (7) and (8) with $\alpha = 30$, $\gamma = 40$

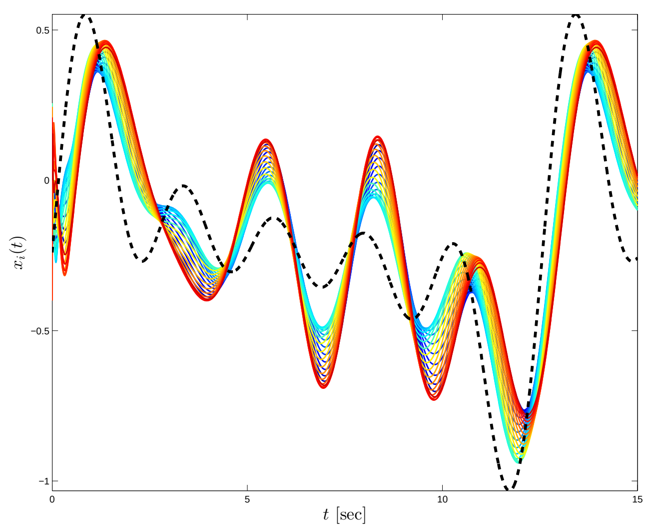

Figure 3. Response of the networked multiagent system under disturbance without implementing disturbance observer (solid lines denote agent states and dashed line denotes the average of applied time–varying inputs).

and $\sigma = 0.1/\gamma$, under disturbance without implementing disturbance observer.

For the disturbance observer given by (13), (14), and (15), without loss of generality, we choose

$$A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}, \tag{44}$$

$C = [1 \quad 0]$ for all agents, i.e., $A_i = A$ and $C_i = C$, $i = 1, \ldots, 25$ so that (A, C) is observable, and with $K_i = K = [67.5 \quad 181.5]^{\mathrm{T}}$, the matrix $(A - KC)$ is Hurwitz.

Under the disturbance observer given by (13), (14), and (15), as expected from Theorem 1, all agents closely track the average of applied time-varying exogenous inputs as presented in Figure 4. Figure 5 also shows that the disturbance estimation error approaches to a neighborhood of zero.

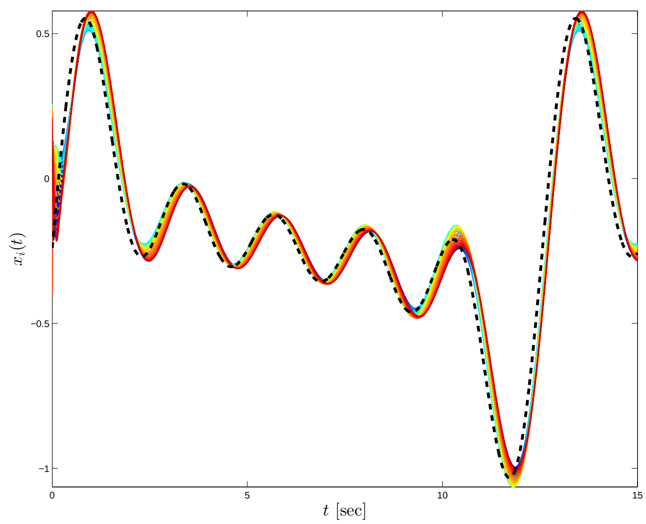

Figure 4. Response of the networked multiagent system under disturbance with implementing disturbance observer (solid lines denote agent states and dashed line denotes the average of applied time–varying inputs).

Example 2: In this example, we consider a network system with 2 active agents and 3 passive agents exchanging information under the connected, undirected graph $\mathcal{G}$ as depicted in Figure 6. The dynamics of each agent is given by

$$\dot{\zeta}_i(t) = A_{\mathrm{m}}\zeta_i(t) + B_{\mathrm{m}}v_i(t), \quad \zeta_i(0) = \zeta_{i0}, \quad i = 1, 2, \ldots, 5, \quad t \geq 0 \tag{45}$$

where $\zeta_i(t) = [\zeta_{x_i} \quad \zeta_{y_i} \quad \zeta_{z_i}]^{\mathrm{T}} \in \mathbb{R}^3$, $t \geq 0$, is the state vector of agent i, with ζ_{x_i}, ζ_{y_i}, ζ_{z_i}, $t \geq 0$, representing the position, velocity and acceleration, respectively. In addition, $v_i(t)$ is the control input, and

$$A_{\mathrm{m}} = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_{\mathrm{m}} = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}. \tag{46}$$

We propose a two-level control hierarchy,[24] which consists of a lower level controller for command tracking and a higher level controller for position consensus of the five agents given by (45). For the lower level

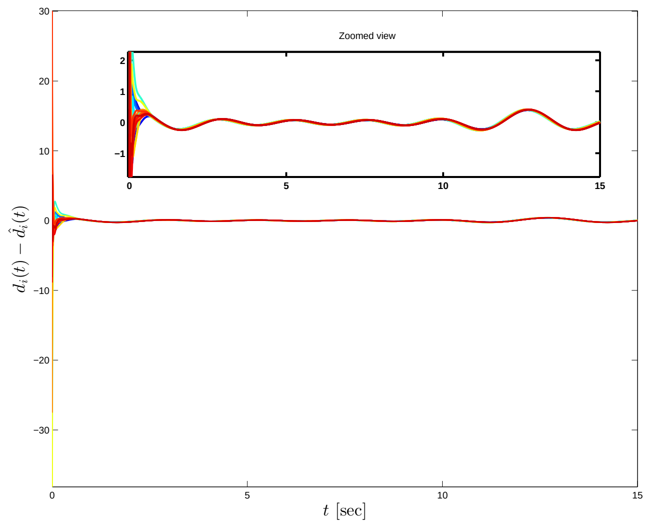

Figure 5. The difference between the estimated disturbance and the disturbance.

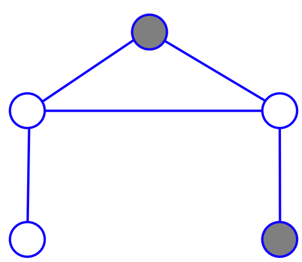

Figure 6. Communication graph of 5 agents (lines denote communication links, gray circles denote active agents, and white circles denote passive agents).

controller, let $x_i(t)$, $i = 1, 2, \ldots, 5$, $t \geq 0$, be the guidance command generated by (7), (8) , (13), (14), and (15), and let $s_i(t)$ denotes the integrator state such that

$$\dot{s}_i(t) = E_{\mathrm{m}}\zeta_i(t) - x_i(t), \quad s_i(0) = q_{i0}, \quad i = 1, 2, \ldots, 5, \quad t \geq 0 \tag{47}$$

where $E_{\mathrm{m}} = [1 \quad 0 \quad 0]$. We define the augmented state as $\bar{\zeta}_i(t) \triangleq [\zeta_i(t)^T \quad s_i(t)]^{\mathrm{T}}$. From (45) and (47), we have

$$\dot{\bar{\zeta}}_i(t) = \bar{A}_{\mathrm{m}}\bar{\zeta}_i(t) + \bar{B}_{\mathrm{m}1}v_i(t) + \bar{B}_{\mathrm{m}2}x_i(t), \quad \bar{\zeta}_i(0) = \bar{\zeta}_{i0}, \quad i = 1, 2, \ldots, 5, \quad t \geq 0 \tag{48}$$

where

$$\bar{A}_{\mathrm{m}} = \begin{bmatrix} A_{\mathrm{m}} & 0 \\ E_{\mathrm{m}} & 0 \end{bmatrix}, \quad \bar{B}_{\mathrm{m}1} = \begin{bmatrix} B_m \\ 0 \end{bmatrix}, \quad \bar{B}_{\mathrm{m}2} = \begin{bmatrix} 0 \\ -\mathrm{I} \end{bmatrix}. \tag{49}$$

Let the control input be given by

$$v_i(t) = -K_{\mathrm{m}}\bar{\zeta}_i(t), \tag{50}$$

where $K_{\mathrm{m}} = [2.2545 \quad 3.2699 \quad 1.9905 \quad 1.4142]$ is designed based on an optimal linear quadratic regulator.

For the higher level controller, we assume that each agent is subjected to a disturbance in the form $d_i = b_i \sin(2t + p_i)$ where b_i and p_i are generated randomly for $i = 1, \ldots, 5$. Through the active–passive network consensus (7), (8) and disturbance observer architecture given by (13), (14), and (15), $x_i(t)$ is generated.

We first consider active agents are subjected to random and isolated constant exogenous inputs. Figure 7 shows the responses of the higher level commands and the positions of lower level controllers under disturbance without implementing disturbance observer. Figure 8 shows that when the disturbance observer is implemented, the higher level controllers are able to attenuate the disturbance and converge to the average of applied constant exogenous inputs while the lower level controllers are able to track the higher level.

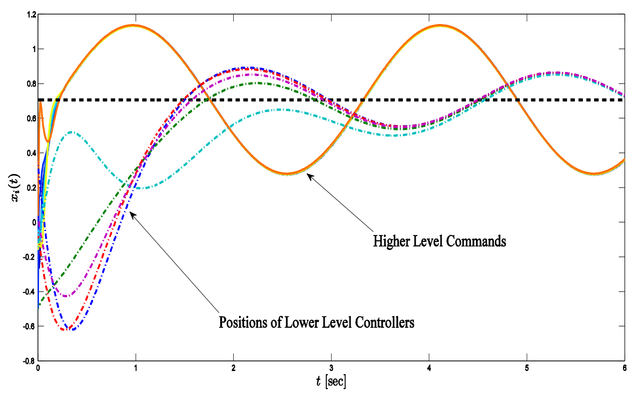

Figure 7. Response of the networked multiagent system under disturbance without implementing disturbance observer (solid lines denote higher level commands, dashed line denotes the average of applied constant exogenous inputs of the higher level controller and dashdot lines denote the positions of lower level controllers).

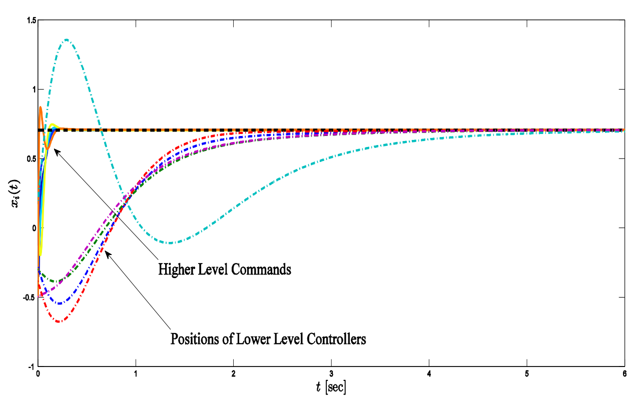

Figure 8. Response of the networked multiagent system under disturbance with implementing disturbance observer (solid lines denote higher level commands, dashed line denotes the average of applied constant exogenous inputs of the higher level controller and dashdot lines denote the positions of lower level controllers).

Now, we consider active agents are subjected to time-varying exogenous inputs given by $c_1(t) = -0.59 + 0.5\sin(0.125t)$, $c_2(t) = 2.0 + 0.5\sin(0.25t)$. Figure 9 shows the responses of both higher and lower level controllers under disturbance without implementing disturbance observer. Again, Figure 10 shows that when the disturbance observer is implemented, the higher level controllers are able to attenuate the disturbance and converge to the average of applied time-varying exogenous inputs while the lower level controllers are able to track the higher level closely. Note that the ability to track the guidance command of the lower level depends on the bandwidth of the system.

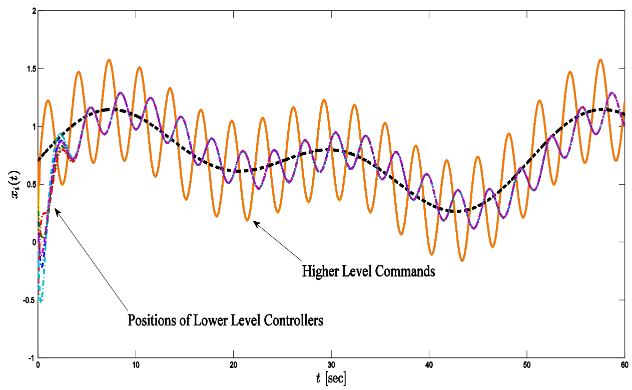

Figure 9. Response of the networked multiagent system under disturbance without implementing disturbance observer (solid lines denote higher level commands, dashed line denotes the average of applied time-varying exogenous inputs of the higher level controller and dashdot lines denote the positions of lower level controllers).

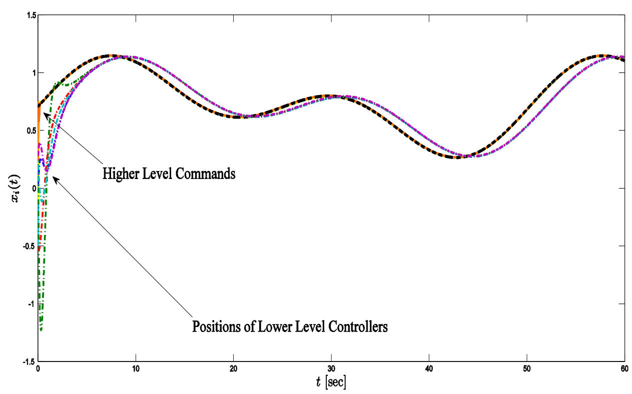

Figure 10. Response of the networked multiagent system under disturbance with implementing disturbance observer (solid lines denote higher level commands, dashed line denotes the average of applied time-varying exogenous inputs of the higher level controller and dashdot lines denote the positions of lower level controllers).

VI. Conclusion

Active-passive networked multiagent systems utilize a novel form of dynamic consensus filters, and they have broad practical applications including, for example, real-time situation awareness and data gathering using sensor networks and distributed control of multirobot systems. For contributing our previous studies in this class of networked multiagent systems, we considered a practical scenario, where active and passive agents are subject to exogenous constant and/or harmonic disturbances. Specifically, for improving resiliency and performance of this class of networked multiagent systems under such persistent disturbances, we utilized a disturbance observer-based approach and showed that the proposed methodology can effectively suppress the effects of exogenous disturbances. Illustrative numerical examples showed the efficacy of the proposed methodology.

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