# Distributed Input and State Estimation using Local Information in Heterogeneous Sensor Networks*

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***Abstract*— A distributed input and state estimation architecture is introduced and analyzed for heterogeneous sensor networks. Specifically, nodes of a given sensor network are allowed to have heterogeneous information roles in the sense that a subset of nodes can be active (that is, subject to observations of a process of interest) and the rest can be passive (that is, subject to no observations). In addition, these nodes are allowed to have nonidentical sensor modalities under the common underlying assumption that they have complimentary properties distributed over the sensor network to achieve collective observability. The key feature of our framework is that it utilizes local information not only during the execution of the proposed distributed input and state estimation architecture but also in its design in that global stability is guaranteed once each node satisfies given local stability conditions independent from the graph topology and neighboring information of these nodes. Several illustrative numerical examples are also provided to demonstrate the efficacy of the proposed architecture.**

## I. INTRODUCTION

As technological advances have boosted the development of integrated microsystems that combine sensing, computing, and communication on a single platform, we are rapidly moving toward a future in which large numbers of integrated microsensors will have the capability to operate in both civilian and military environments. Such large-scale sensor networks will support applications with dramatically increasing levels of complexity including situational awareness, environment monitoring, scientific data gathering, collaborative information processing, and search and rescue; to name but a few examples. One of the important areas of research in sensor networks is the development of distributed estimation algorithms for dynamic information fusion. Because, these algorithms are reliable to possible loss of a subset of nodes and communication links and they are flexible in the sense that nodes can be added and removed by making only local changes to the sensor network. Although distributed estimation algorithms have had strong appeal for these reasons, a critical roadblock to achieve correct and reliable dynamic information fusion with these algorithms is heterogeneity.

Heterogeneity in sensor networks is unavoidable in real-world applications. To elucidate this fact, consider a target estimation problem as a motivating example. Specifically, nodes of a given sensor network can have heterogeneous information roles in this example such that a subset of nodes can be subject to observations of this target (active nodes) and the rest can be subject to no observations (passive nodes). Thus, during the dynamic information fusion process, only active nodes have to be taken into account. In addition, note that nodes of a sensor network can also have nonidentical sensor modalities; for example, a subset of nodes can sense the target position and others can sense the target velocity. This case needs to be considered in the dynamic information fusion process as well.

Dealing with these classes of heterogeneity in sensor networks to achieve correct and reliable dynamic information fusion is a challenging task using distributed estimation algorithms. Toward this end, notable contributions in the literature include [1]–[20]. Specifically, the authors of [1]–[8] propose dynamic consensus algorithms that are suitable for sensor networks with all nodes being active. However, as discussed earlier, a subset of nodes in a sensor network can be passive in that they may not be able to sense a process of interest and collect information. While the authors of [9]–[11] present methods that cover specific applications when a subset of nodes are passive (and the remaining nodes are active), their results are in the context of static consensus, and hence, they are not suitable in their presented form for dynamic data-driven applications.

The authors of [12]–[18] introduced the concept of sensor networks with active and passive nodes in the context of dynamic consensus. However, nodes of the considered class of sensor networks are implicitly assumed to have identical sensor modalities since each node is modeled using single integrator dynamics. Finally, the authors of [19] and [20] consider dynamic information fusion for sensor networks having nonidentical sensor modalities, where the former contribution requires each node to be active via sensing some states of a process of interest. While this is implicitly not assumed in the latter contribution, global information is required during the distributed algorithm design in terms of guaranteeing global stability — although the proposed algorithm can be executed by solely relying on local information exchange between neighboring nodes.

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The contribution of this paper is to introduce and analyze a distributed input and state estimation architecture for heterogeneous sensor networks. Specifically, nodes of a given sensor network are allowed to have heterogeneous information roles in the sense that a subset of nodes can be active (that is, subject to observations of a process of interest) and the rest can be passive (that is, subject to no observations). In addition, these nodes are allowed to have nonidentical sensor modalities under the common underlying assumption that they have complimentary properties distributed over the sensor network to achieve collective observability (see, for example, [19] and [20], and references therein).

The key feature of our framework is that it utilizes local information not only during the execution of the proposed distributed input and state estimation architecture but also in its design unlike the results in [20]; that is, global stability is guaranteed once each node satisfies given local stability conditions independent from the graph topology and neighboring information of these nodes. As it is standard in the classical input and state estimation literature (see, for example, [21]–[25], and references therein), it should be also noted that we do not make a passivity or passivity-like assumption by resorting to a similar in spirit idea from [26]–[31]. Several illustrative numerical examples are also provided to demonstrate the efficacy of the proposed architecture.

The organization of this paper is as follows. Section II introduces necessary mathematical preliminaries to develop the main results of this paper. System-theoretic design and analysis of the proposed distributed input and state estimation architecture are given in Section III, where Section IV presents several illustrative numerical examples. Finally, concluding remarks are summarized in Section V.

## II. Mathematical Preliminaries

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrices, $\mathbf{1}_n$ denotes the $n \times 1$ vector of all ones, and $\mathrm{I}_n$ denotes the $n \times n$ identity matrix. In addition, we write $(\cdot)^{\mathrm{T}}$ for transpose, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ for the minimum and maximum eigenvalue of the Hermitian matrix A, respectively, $\lambda_i(A)$ for the *i*-th eigenvalue of A, where A is symmetric and the eigenvalues are ordered from least to greatest value, $\mathrm{diag}(a)$ for the diagonal matrix with the vector $a$ on its diagonal, $[x]_i$ for the entry of the vector $x$ on the *i*-th row, and $[A]_{ij}$ for the entry of the of the matrix A on the *i*-th row and *j*-th column.

We now recall some basic notions from graph theory and refer to textbooks [32] and [33] for details. Specifically, graphs are broadly adopted in the sensor networks literature to encode interactions between nodes. An undirected graph $\mathcal{G}$ is defined by a set $\mathcal{V}_{\mathcal{G}} = \{1, \ldots, N\}$ of nodes and a set $\mathcal{E}_{\mathcal{G}} \subset \mathcal{V}_{\mathcal{G}} \times \mathcal{V}_{\mathcal{G}}$ of edges. If $(i, j) \in \mathcal{E}_{\mathcal{G}}$, then the nodes $i$ and $j$ are neighbors and the neighboring relation is indicated with $i \sim j$. The degree of a node is given by the number of its neighbors. Letting $d_i$ be the degree of node $i$, then the degree matrix of a graph $\mathcal{G}$, $\mathcal{D}(\mathcal{G}) \in \mathbb{R}^{N\times N}$, is given by

$$\mathcal{D}(\mathcal{G}) \triangleq \mathrm{diag}(d), \quad d = [d_1, \ldots, d_N]^{\mathrm{T}}. \tag{1}$$

A path $i_0 i_1 \ldots i_L$ is a finite sequence of nodes such that $i_{k-1} \sim i_k$, $k = 1, \ldots, L$, and a graph $\mathcal{G}$ is connected if there is a path between any pair of distinct nodes. The adjacency matrix of a graph $\mathcal{G}$, $\mathcal{A}(\mathcal{G}) \in \mathbb{R}^{N\times N}$, is given by

$$[\mathcal{A}(\mathcal{G})]_{ij} \quad \triangleq \quad \begin{cases} 1, & \text{if } (i,j) \in \mathcal{E}_{\mathcal{G}}, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The Laplacian matrix of a graph, $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_+^{N\times N}$, playing a central role in many graph-theoretic treatments of sensor networks, is given by

$$\mathcal{L}(\mathcal{G}) \quad \triangleq \quad \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G}). \tag{3}$$

The spectrum of the Laplacian of an undirected and connected graph can be ordered as

$$0 \quad = \quad \lambda_1(\mathcal{L}(\mathcal{G})) < \lambda_2(\mathcal{L}(\mathcal{G})) \le \cdots \le \lambda_N(\mathcal{L}(\mathcal{G})), \tag{4}$$

with $\mathbf{1}_N$ as the eigenvector corresponding to the zero eigenvalue $\lambda_1(\mathcal{L}(\mathcal{G}))$ and $\mathcal{L}(\mathcal{G})\mathbf{1}_N = \mathbf{0}_N$ and $\mathrm{e}^{\mathcal{L}(\mathcal{G})}\mathbf{1}_N = \mathbf{1}_N$. Throughout this paper, we assume that the graph $\mathcal{G}$ of a given sensor network is undirected and connected.

Finally, the following lemmas are necessary to develop the main results of this paper.

**Lemma 1** (Proposition 8.1.2, [34])**.** Let $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times n}$. If $A \ge 0$ and $B > 0$, then $A + B > 0$.

**Lemma 2** (Proposition 8.2.4, [34])**.** Let $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times m}$, and

$$X = \begin{bmatrix} A & B \\ B^{\mathrm{T}} & C \end{bmatrix}.$$

Then, $X \ge 0$ if and only if one or both of the conditions given by

$$A \ge 0, \quad C - B^{\mathrm{T}}A^{\dagger}B \ge 0, \quad (I - AA^{\dagger})B = 0, \tag{5}$$

$$C \ge 0, \quad A - BC^{\dagger}B^{\mathrm{T}} \ge 0, \quad (I - CC^{\dagger})B^{\mathrm{T}} = 0, \tag{6}$$

hold.

## III. Distributed Input and State Estimation

In this section, we introduce and analyze a distributed input and state estimation architecture for heterogeneous sensor networks. For this purpose, consider a process of interest with the dynamics given by

$$\dot{x}(t) = Ax(t) + Bw(t), \quad x(0) = x_0, \tag{7}$$

where $x(t) \in \mathbb{R}^n$ denotes the unmeasurable process state vector, $w(t) \in \mathbb{R}^p$ denotes an unknown bounded input to this process with a bounded time rate of change, $A \in \mathbb{R}^{n\times n}$ denotes a Hurwitz system matrix, and $B \in \mathbb{R}^{n\times p}$ denotes the system input matrix.

Next, consider a sensor network with $N$ nodes exchanging information among each other using their local measurements according to an undirected and connected graph $\mathcal{G}$. In the sense of [12]–[18], if a node $i$, $i=1,\dots,N$, is subject to observations of the process (7) given by

$$y_i(t) = C_i x(t), \tag{8}$$

where $y_i(t) \in \mathbb{R}^p$ and $C_i \in \mathbb{R}^{p\times n}$ denote the measurable process output and the system output matrix for node $i$, $i=1,\dots,N$, respectively, then we say that it is an active node. Similarly, if a node $i$, $i=1,\dots,N$, has no observations, then we say that it is a passive node. Notice that the above formulation allows for nonidentical sensor modalities since $C_i$ of active nodes can be different. Here, as standard, we assume that each active node has complimentary properties distributed over the sensor network to guarantee collective observability (see, for example, [19] and [20], and references therein), although the pairs $(A, C_i)$, $i=1,\dots,N$, may not be locally observable.

In this paper, we are interested in the problem of distributively estimating the unmeasurable state $x(t)$ and the unknown input $w(t)$ of the process given by (7) using a sensor network, where active nodes are subject to the observations given by (8). For this purpose, the rest of this section is divided into two parts, where we first introduce the proposed distributed estimation architecture and then analyze its stability in detail.

### A. Proposed Distributed Estimation Architecture

For node $i$, $i=1,\dots,N$, consider the estimation algorithm given by

$$\begin{aligned}\dot{\hat{x}}_i(t) &= (A-\gamma P_i^{-1})\hat{x}_i(t) + B\hat{w}_i(t) + g_i L_i\big(y_i(t) \\ &\quad -C_i\hat{x}_i(t)\big) - \alpha P_i^{-1}\sum_{i\sim j}(\hat{x}_i(t)-\hat{x}_j(t)), \\ &\qquad \hat{x}_i(0)=\hat{x}_{i0}, \end{aligned}\tag{9}$$

$$\begin{aligned}\dot{\hat{w}}_i(t) &= g_i K_i(y_i(t)-C_i\hat{x}_i(t)) - (\sigma_i K_i + \gamma \mathrm{I}_p)\hat{w}_i(t) \\ &\quad -\alpha\sum_{i\sim j}(\hat{w}_i(t)-\hat{w}_j(t)), \quad \hat{w}_i(0)=\hat{w}_{i0}, \end{aligned}\tag{10}$$

where $\hat{x}_i(t)\in\mathbb{R}^n$ is a local state estimate of $x(t)$ for node $i$, $\hat{w}_i\in\mathbb{R}^p$ is a local input estimate of $w(t)$ for node $i$, $L_i\in\mathbb{R}^{n\times p}$ and $K_i\in\mathbb{R}^{p\times p}$ are design matrices of node $i$ with $K_i$ being symmetric and positive definite, and $\alpha$, $\gamma$, and $\sigma_i\in\mathbb{R}$ are positive design coefficients for node $i$. Here, $g_i=1$ for active nodes and otherwise $g_i=0$. In addition, $P_i>0$ is the consensus gain satisfying the linear matrix inequality given by

$$R_i = \begin{bmatrix} \bar{A}_i^{\mathrm{T}}P_i + P_i\bar{A}_i & -P_iB + g_iC_i^{\mathrm{T}}K_i^{\mathrm{T}} \\ -B^{\mathrm{T}}P_i + g_iK_iC_i & -2\sigma_iK_i\end{bmatrix} \leq 0, \tag{11}$$

where

$$\bar{A}_i \triangleq A - g_iL_iC_i. \tag{12}$$

**Remark 1.** The local condition given by (11) for node $i$, $i=1,\dots,N$, plays a central role in the stability analysis presented in the next section. Specifically, if the proposed input and state estimation architecture given by (9) and (10) satisfies the local condition given by (11) for each node, then stability is guaranteed for the overall sensor network globally. In addition, note that the local condition given by (11) is well-posed. To see this, for example, let $P_i$ satisfy the linear matrix inequality given by $\bar{A}_i^{\mathrm{T}}P_i + P_i\bar{A}_i < 0$, $i=1,\dots,N$. Then it can be readily shown that there exists a sufficiently large $\sigma_i$, $i=1,\dots,N$, such that (11) holds. As a special case, if all nodes are active and a well-known condition $P_iB = C_i^{\mathrm{T}}K_i^{\mathrm{T}}$ holds (see, for example, [21]–[25], and references therein), then it can be easily seen that (11) holds even for sufficiently small values of $\sigma_i$, $i=1,\dots,N$. Similarly, for the same special case when all nodes are active, if $\mathcal{H}(s) \triangleq K_iC_i(s\mathrm{I}-\bar{A}_i)^{-1}B + \sigma_iK_i$ is passive, $i=1,\dots,N$, then (11) is feasible and vice versa [35].

### B. Stability Analysis

Let $\tilde{x}_i(t) \triangleq x(t)-\hat{x}_i(t)$ and $\tilde{w}_i(t)\triangleq \hat{w}_i(t)-w(t)$. Then, based on (9) and (10), we have

$$\begin{aligned}\dot{\tilde{x}}_i(t) &= Ax(t)+Bw(t)-(A-\gamma P_i^{-1})\hat{x}_i(t) \\ &\quad -B\hat{w}_i(t) - g_iL_i(y_i(t)-C_i\hat{x}_i(t)) \\ &\quad +\alpha P_i^{-1}\sum_{i\sim j}(\hat{x}_i(t)-\hat{x}_j(t)) \\ &= (A-g_iL_iC_i)\tilde{x}_i(t) - B\tilde{w}_i(t) \\ &\quad +\alpha P_i^{-1}\sum_{i\sim j}(\hat{x}_i(t)-\hat{x}_j(t)) + \gamma P_i^{-1}\hat{x}_i(t) \\ &= \bar{A}_i\tilde{x}_i(t) - B\tilde{w}_i(t) - \alpha P_i^{-1}\sum_{i\sim j}(\tilde{x}_i(t)-\tilde{x}_j(t)) \\ &\quad -\gamma P_i^{-1}\big(\tilde{x}_i(t)-x(t)\big), \quad \tilde{x}_i(0)=\tilde{x}_{i0}, \end{aligned}\tag{13}$$

$$\begin{aligned}\dot{\tilde{w}}_i(t) &= g_iK_iC_i\tilde{x}_i(t) - \sigma_iK_i(\tilde{w}_i(t)+w(t)) \\ &\quad -\alpha\sum_{i\sim j}(\tilde{w}_i(t)-\tilde{w}_j(t)) \\ &\quad -\gamma(\tilde{w}_i(t)+w(t)) - \dot{w}(t), \quad \tilde{w}_i(0)=\tilde{w}_{i0}. \end{aligned}\tag{14}$$

Now, considering the aggregated vectors given by

$$\tilde{x}(t) \triangleq [\tilde{x}_1(t),\tilde{x}_2(t),\dots,\tilde{x}_N(t)] \in \mathbb{R}^{Nn}, \tag{15}$$

$$\tilde{w}(t) \triangleq [\tilde{w}_1(t),\tilde{w}_2(t),\dots,\tilde{w}_N(t)] \in \mathbb{R}^{Np}, \tag{16}$$

we can write the error dynamics in a compact form as

$$\begin{aligned}\dot{\tilde{x}}(t) &= \bar{A}\tilde{x}(t) - (\mathrm{I}_N\otimes B)\tilde{w}(t) - P^{-1}(F\otimes \mathrm{I}_n)\tilde{x}(t) \\ &\quad +\gamma P^{-1}(\mathbf{1}_N\otimes \mathrm{I}_n)x(t), \end{aligned}\tag{17}$$

$$\begin{aligned}\dot{\tilde{w}}(t) &= M\tilde{x}(t) - \bar{K}\big(\tilde{w}(t)+(\mathbf{1}_N\otimes\mathrm{I}_p)w(t)\big) \\ &\quad -(F\otimes\mathrm{I}_p)\tilde{w}(t) - \gamma(\mathbf{1}_N\otimes\mathrm{I}_p)w(t) \\ &\quad -(\mathbf{1}_N\otimes\mathrm{I}_p)\dot{w}(t), \end{aligned}\tag{18}$$

where

$$\bar{A} \triangleq \mathrm{diag}([\bar{A}_1,\bar{A}_2,\dots,\bar{A}_N]), \tag{19}$$

$$M \triangleq \mathrm{diag}([g_1K_1C_1, g_2K_2C_2,\dots,g_NK_NC_N]), \tag{20}$$

$$\bar{K} \triangleq \mathrm{diag}([\sigma_1K_1,\sigma_2K_2,\dots,\sigma_NK_N]), \tag{21}$$

$$F \triangleq \alpha\mathcal{L}(\mathcal{G}) + \gamma\mathrm{I}_N, \tag{22}$$

$$P \triangleq \mathrm{diag}([P_1,P_2,\dots,P_N]), \tag{23}$$

with $\mathcal{L}(\mathcal{G})$ being the Laplacian matrix. Note that $P>0$ follows from $P_i>0$.

**Theorem 1.** Consider the process given by (7) and the distributed input and state estimation architecture given by (9) and (10). Assume (11) holds and nodes exchange information using local measurements subject to an undirected and connected graph $\mathcal{G}$. Then, the error dynamics given by (17) and (18) are uniformly bounded.

*Proof.* Consider the Lyapunov function candidate given by

$$V(\tilde{x},\tilde{w}) = \tilde{x}^{\mathrm{T}} P\tilde{x} + \tilde{w}^{\mathrm{T}}\tilde{w}. \tag{24}$$

Note that $V(0) = 0$ and $V(\tilde{x},\tilde{w}) > 0$ for all $(\tilde{x},\tilde{w}) \neq (0,0)$. Taking time-derivative of $V(\tilde{x},\tilde{w})$ along the trajectories of (17) and (18) yields

$$\begin{aligned}
\dot{V}(\cdot) =\ & \tilde{x}^{\mathrm{T}}(t)(\bar{A}^{\mathrm{T}}P + P\bar{A})\tilde{x}(t) - 2\tilde{x}^{\mathrm{T}}(t)P(\mathrm{I}_N\otimes B)\tilde{w}(t)\\
&-2\tilde{x}^{\mathrm{T}}(t)(F\otimes \mathrm{I}_n)\tilde{x}(t) + 2\gamma\tilde{x}^{\mathrm{T}}(t)(\mathbf{1}_N\otimes \mathrm{I}_n)x(t)\\
&+2\tilde{w}^{\mathrm{T}}(t)M\tilde{x}(t) - 2\tilde{w}^{\mathrm{T}}(t)\bar{K}\tilde{w}(t)\\
&-2\tilde{w}^{\mathrm{T}}(t)(F\otimes \mathrm{I}_p)\tilde{w}(t)\\
&-2\tilde{w}^{\mathrm{T}}(t)(\bar{K}+\gamma \mathrm{I}_{Np})(\mathbf{1}_N\otimes \mathrm{I}_p)w(t)\\
&-2\tilde{w}^{\mathrm{T}}(t)(\mathbf{1}_N\otimes \mathrm{I}_p)\dot{w}(t)\\
=\ & \Big(\tilde{x}^{\mathrm{T}}(t)(\bar{A}^{\mathrm{T}}P + P\bar{A})\tilde{x}(t) - 2\tilde{w}^{\mathrm{T}}(t)\bar{K}\tilde{w}(t)\\
&-2\tilde{x}^{\mathrm{T}}(t)\big(P(\mathrm{I}_N\otimes B) - M^{\mathrm{T}}\big)\tilde{w}(t)\Big)\\
&-2\tilde{x}^{\mathrm{T}}(t)(F\otimes \mathrm{I}_n)\tilde{x}(t) - 2\tilde{w}^{\mathrm{T}}(t)(F\otimes \mathrm{I}_p)\tilde{w}(t)\\
&+2\gamma\tilde{x}^{\mathrm{T}}(t)(\mathbf{1}_N\otimes \mathrm{I}_n)x(t)\\
&-2\tilde{w}^{\mathrm{T}}(t)(\bar{K}+\gamma \mathrm{I}_{Np})(\mathbf{1}_N\otimes \mathrm{I}_p)w(t)\\
&-2\tilde{w}^{\mathrm{T}}(t)(\mathbf{1}_N\otimes \mathrm{I}_p)\dot{w}(t)\\
=\ & \begin{bmatrix}\tilde{x}^{\mathrm{T}}(t) & \tilde{w}^{\mathrm{T}}(t)\end{bmatrix}\\
&\cdot\begin{bmatrix}\bar{A}^{\mathrm{T}}P + P\bar{A} & -P(\mathrm{I}_N\otimes B) + M^{\mathrm{T}}\\ -(\mathrm{I}_N\otimes B^{\mathrm{T}})P + M & -2\bar{K}\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ \tilde{w}(t)\end{bmatrix}\\
&+\begin{bmatrix}\tilde{x}^{\mathrm{T}}(t) & \tilde{w}^{\mathrm{T}}(t)\end{bmatrix}\begin{bmatrix}-2(F\otimes \mathrm{I}_n) & 0\\ 0 & -2(F\otimes \mathrm{I}_p)\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ \tilde{w}(t)\end{bmatrix}\\
&+2\begin{bmatrix}\tilde{x}^{\mathrm{T}}(t) & \tilde{w}^{\mathrm{T}}(t)\end{bmatrix}\\
&\cdot\begin{bmatrix}\gamma(\mathbf{1}_N\otimes \mathrm{I}_n)x(t)\\ -(\bar{K}+\gamma \mathrm{I}_{Np})(\mathbf{1}_N\otimes \mathrm{I}_p)w(t) - (\mathbf{1}_N\otimes \mathrm{I}_p)\dot{w}(t)\end{bmatrix}\\
=\ & z^{\mathrm{T}}(t)R_A z(t) + z^{\mathrm{T}}(t)R_B z(t) + 2z(t)\phi\\
=\ & z^{\mathrm{T}}(t)Rz(t) + 2z(t)\phi,
\end{aligned} \tag{25}$$

where

$$z(t) \triangleq [\tilde{x}^{\mathrm{T}}(t),\ \tilde{w}^{\mathrm{T}}(t)]^{\mathrm{T}}, \tag{26}$$

$$R_A \triangleq \begin{bmatrix}\bar{A}^{\mathrm{T}}P + P\bar{A} & -P(\mathrm{I}_N\otimes B) + M^{\mathrm{T}}\\ -(\mathrm{I}_N\otimes B^{\mathrm{T}})P + M & -2\bar{K}\end{bmatrix}, \tag{27}$$

$$R_B \triangleq \begin{bmatrix}-2(F\otimes \mathrm{I}_n) & 0\\ 0 & -2(F\otimes \mathrm{I}_p)\end{bmatrix}, \tag{28}$$

$$\begin{aligned}R &\triangleq R_A + R_B\\ &= \begin{bmatrix}\bar{A}^{\mathrm{T}}P + P\bar{A} - 2(F\otimes \mathrm{I}_n) & -P(\mathrm{I}_N\otimes B) + M^{\mathrm{T}}\\ -(\mathrm{I}_N\otimes B^{\mathrm{T}})P + M & -2\bar{K} - 2(F\otimes \mathrm{I}_p)\end{bmatrix},\end{aligned} \tag{29}$$

$$\phi \triangleq \begin{bmatrix}\gamma(\mathbf{1}_N\otimes \mathrm{I}_n)x(t)\\ -(\bar{K}+\gamma \mathrm{I}_{Np})(\mathbf{1}_N\otimes \mathrm{I}_p)w(t) - (\mathbf{1}_N\otimes \mathrm{I}_p)\dot{w}(t)\end{bmatrix}. \tag{30}$$

Note that $(F\otimes \mathrm{I}_n) > 0$ and $(F\otimes \mathrm{I}_p) > 0$ follows from $F > 0$, and hence, $R_B < 0$.

Next, since the linear matrix inequality given by (11) holds, it follows that

$$\bar{A}_i^{\mathrm{T}}P_i + P_i\bar{A}_i \leq 0, \tag{31}$$

$$\begin{aligned}N_i \triangleq\ & -2\sigma_i K - (-B^{\mathrm{T}}P_i + g_iKC_i)(\bar{A}_i^{\mathrm{T}}P_i + P_i\bar{A}_i)^{\dagger}\\ &\cdot(-P_iB + g_iC_i^{\mathrm{T}}K^{\mathrm{T}}) \leq 0,\end{aligned} \tag{32}$$

$$\begin{aligned}Q_i \triangleq\ & \big(\mathrm{I}_n - (\bar{A}_i^{\mathrm{T}}P_i + P_i\bar{A}_i)(\bar{A}_i^{\mathrm{T}}P_i + P_i\bar{A}_i)^{\dagger}\big)\\ &\cdot(-P_iB + g_iC_i^{\mathrm{T}}K^{\mathrm{T}}) = 0.\end{aligned} \tag{33}$$

by applying Lemma 2 to (11). Note that

$$\bar{A}^{\mathrm{T}}P + P\bar{A} = \begin{bmatrix}\hat{A}_1 & 0 & \cdots & 0\\ 0 & \hat{A}_2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \hat{A}_N\end{bmatrix} \leq 0, \tag{34}$$

as a consequence of (31), where $\hat{A}_i \triangleq \bar{A}_i^{\mathrm{T}}P_i + P_i\bar{A}_i$ for $i = 1,\ldots,N$. In addition, it follows from (32) that

$$\begin{aligned}N \triangleq\ & -2\bar{K} - \big(-(\mathrm{I}_N\otimes B^{\mathrm{T}})P + M\big)(\bar{A}^{\mathrm{T}}P + P\bar{A})^{\dagger}\\ &\cdot(-P(\mathrm{I}_N\otimes B) + M^{\mathrm{T}})\\ =\ & \begin{bmatrix}N_1 & 0 & \cdots & 0\\ 0 & N_2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & N_N\end{bmatrix} \leq 0,\end{aligned} \tag{35}$$

holds. Finally, (33) leads to

$$\begin{aligned}Q \triangleq\ & \big(\mathrm{I}_{Nn} - (\bar{A}^{\mathrm{T}}P + P\bar{A})(\bar{A}^{\mathrm{T}}P + P\bar{A})^{\dagger}\big)\\ &\cdot(-P(\mathrm{I}_N\otimes B) + M^{\mathrm{T}})\\ =\ & \begin{bmatrix}Q_1 & 0 & \cdots & 0\\ 0 & Q_2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & Q_N\end{bmatrix} = 0.\end{aligned} \tag{36}$$

Now, by Lemma 2, $R_A \leq 0$ as a direct consequence of (34), (35) and (36). Thus, by Lemma 1, $R = R_A + R_B < 0$.

Note that $\|x(t)\|_2 \leq \bar{x}$ (that follows from A being a Hurwitz matrix), $\|w(t)\|_2 \leq \bar{w}$, and $\|\dot{w}(t)\|_2 \leq \bar{\dot{w}}$. Therefore, $\|\phi\|_2 \leq \bar{\phi}$ with

$$\begin{aligned}\bar{\phi} \triangleq \Big(&\gamma^2\|(\mathbf{1}_N\otimes \mathrm{I}_n)\|_2^2\bar{x}^2 + \|\bar{K}+\gamma \mathrm{I}_{Np}\|_2^2\|\mathbf{1}_N\otimes \mathrm{I}_p\|_2^2\bar{w}^2\\ &+\|\mathbf{1}_N\otimes \mathrm{I}_p\|_2^2\bar{\dot{w}}^2\Big)^{1/2}.\end{aligned} \tag{37}$$

Now, one can write

$$\begin{aligned}\dot{V}(\cdot) &= z^{\mathrm{T}}(t)Rz(t) + 2z^{\mathrm{T}}(t)\phi\\ &\leq \lambda_{\max}(R)\|z(t)\|_2^2 + 2\|z(t)\|_2\bar{\phi},\end{aligned} \tag{38}$$

with $\lambda_{\max}(R) < 0$. Letting $\mu \triangleq \frac{-2\bar{\phi}}{\lambda_{\max}(R)} > 0$ and $\Omega \triangleq \{z(t) : \|z(t)\|_2 \leq \mu\}$, it follows that $\dot{V}(\cdot) < 0$ outside the compact set $\Omega$, and hence, the error dynamics given by (17) and (18) are uniformly bounded [36], [37]. ■

The following corollary to the above theorem is now immediate.

**Corollary 1.** Consider the process given by (7) and the distributed input and state estimation architecture given by (9) and (10). Assume (11) holds and nodes exchange information using local measurements subject to an undirected and connected graph $\mathcal{G}$. Then, the bounds

$$\|\tilde{x}(t)\|_2 \leq \sqrt{\frac{\lambda_{\min}(\bar{P})\mu^2}{\lambda_{\min}(P)}} \triangleq \psi, \quad (39)$$

$$\|\tilde{w}(t)\|_2 \leq \sqrt{\lambda_{\max}(\bar{P})\mu^2} \triangleq \zeta, \quad (40)$$

hold for $t \geq T$, where

$$\bar{P} = \begin{bmatrix} P & 0 \\ 0 & \mathrm{I}_{Np} \end{bmatrix}. \quad (41)$$

*Proof.* Note that

$$\begin{aligned} V(\cdot) &= \tilde{x}^{\mathrm{T}}(t)P\tilde{x}(t) + \tilde{w}^{\mathrm{T}}(t)\tilde{w}(t) \\ &= \begin{bmatrix} \tilde{x}^{\mathrm{T}}(t) & \tilde{w}^{\mathrm{T}}(t) \end{bmatrix} \begin{bmatrix} P & 0 \\ 0 & \mathrm{I}_{Np} \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \tilde{w}(t) \end{bmatrix} \\ &= z^{\mathrm{T}}(t)\bar{P}z(t). \quad (42) \end{aligned}$$

In the proof of Theorem 1, we show that $V(\cdot)$ cannot grow outside the compact set $\Omega$, thus (39) follows from $\lambda_{\min}(P)\|\tilde{x}(t)\|_2^2 \leq V\big(\tilde{x}(t), \tilde{w}(t)\big) \leq \lambda_{\max}(\bar{P})\|\tilde{z}(t)\|_2^2 \leq \lambda_{\max}(\bar{P})\mu^2$. Identically, (40) follows from $\|\tilde{w}(t)\|_2^2 \leq V\big(\tilde{x}(t), \tilde{w}(t)\big) \leq \lambda_{\max}(\bar{P})\|\tilde{z}(t)\|_2^2 \leq \lambda_{\max}(\bar{P})\mu^2$. The proof is now complete. ■

**Remark 2.** Since the ultimate bounds given by (39) and (40) depend on the design parameters of the proposed distributed input and state estimation architecture, they can be used as design metrics such that the design parameters can be judiciously selected to make (39) and (40) small. However, unlike the stability of our framework that is guaranteed once each node satisfies the local condition given by (11), such a performance characterization requires global information. However, one can further analyze the effect of each specific design parameter to these ultimate bounds and make conclusions without possibly requiring global information, which will be considered as a future research direction.

**Remark 3.** Note that the terms "$-\gamma P_i^{-1}\hat{x}_i(t)$" and "$-(\sigma_i K_i + \gamma \mathrm{I}_p)\hat{w}_i(t)$" appearing respectively in (9) and (10) are often referred to as leakage terms. If the gains "$\gamma P_i^{-1}$" and "$\sigma_i K_i + \gamma \mathrm{I}_p$" respectively multiplying these terms are not small, then they can result in poor performance (see, for example, [38], [39] and references therein), and hence, it is of common practice to choose these multiplier gains to be small. However, as noted in Remark 1, $\sigma_i$ may not be chosen as sufficiently small unless all nodes are active and the condition $P_i B = C_i^{\mathrm{T}} K_i^{\mathrm{T}}$ holds. Therefore, we cast (11) as an optimization problem given by

$$\text{minimize } \sigma_i, \quad (43)$$
$$\text{subject to } (11), \quad (44)$$

for all nodes $i = 1, \ldots, N$.

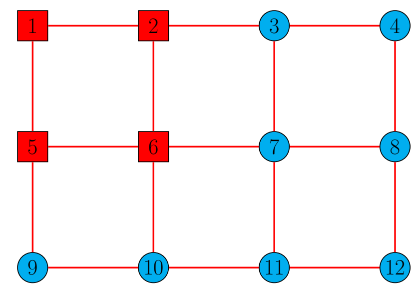


Fig. 1. Communication graph of the sensor network in Example 1 with 4 active nodes and 8 pasive nodes (lines denote communication links, squares denote active nodes, and circles denote passive nodes).

## IV. Illustrative Numerical Examples

In this section, we present several numerical examples to illustrate the results discussed in Sections III.A and III.B. For this purpose, consider a process composed of two decoupled systems with the dynamics given by (7), where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_{\mathrm{n}1}^2 & -2\omega_{\mathrm{n}1}\xi_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_{\mathrm{n}2}^2 & -2\omega_{\mathrm{n}2}\xi_2 \end{bmatrix}, \quad (45)$$

$$B = \begin{bmatrix} 0 & 0 \\ \omega_{\mathrm{n}1}^2 & 0 \\ 0 & 0 \\ 0 & \omega_{\mathrm{n}2}^2 \end{bmatrix}, \quad (46)$$

$\omega_{\mathrm{n}1} = 1.2$, $\xi_1 = 0.9$, $\omega_{\mathrm{n}2} = 0.5$, and $\xi_2 = 0.6$. This process, for example, can represent a linearized simple vehicle model with the first and third states corresponding to the positions in the $x$ and $y$ directions, respectively, while the second and fourth states corresponding to the velocities in the $x$ and $y$ directions, respectively. The initial conditions are set to $x_0^{\mathrm{T}} = [-3,\ 0.5,\ 2.5,\ 0.25]$. In addition, we consider the input given by

$$w(t) = \begin{bmatrix} 2.5\sin(t) \\ 4\cos(1.2t) \end{bmatrix}. \quad (47)$$

**Example 1.** For the first example, we consider a sensor network with 12 nodes exchanging information over an undirected and connected graph topology, where there are 4 active nodes and 8 passive nodes as shown in Figure 1. Each node's sensing capability is represented by (8) with the output matrices

$$C_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad (48)$$

for the odd index nodes and

$$C_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (49)$$

for the even index nodes. In addition, all nodes are subject to zero initial conditions and we set $K_i = \mathrm{diag}([100; 100])$,

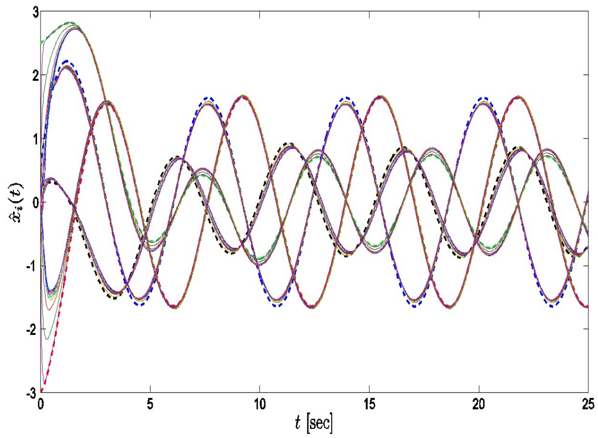


Fig. 2. State estimates of the sensor network in Example 1 with 4 active nodes and 8 passive nodes under the proposed architecture (9) and (10) (the dash lines denote the states of the actual process and the solid lines denote the state estimates of nodes).

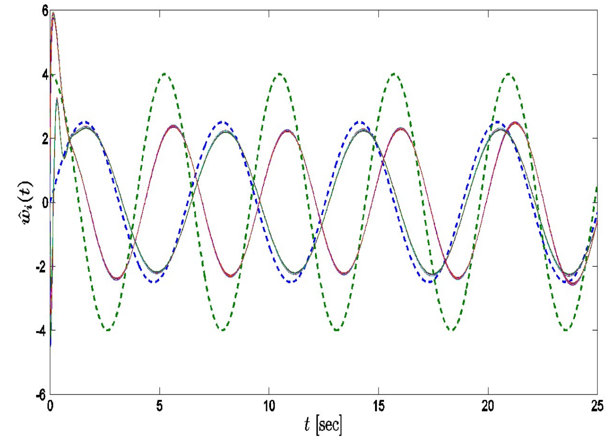


Fig. 3. Input estimates of the sensor network in Example 1 with 4 active nodes and 8 passive nodes under the proposed architecture (9) and (10) (the dash lines denote the inputs of the actual process and the solid lines denote the input estimates of nodes).

$\alpha = 50$, and $\gamma = 0.1$. For the observer gain $L_i$, the odd index nodes are subject to

$$L_i = \begin{bmatrix} 18.969 & -1.907 \\ -0.487 & -0.076 \\ -1.939 & 19.130 \\ -0.285 & 2.492 \end{bmatrix}, \tag{50}$$

while the even index nodes are subject to

$$L_i = \begin{bmatrix} -2.388 & 0.358 \\ 5.831 & -0.804 \\ 0.428 & -2.398 \\ -1.038 & 6.765 \end{bmatrix}. \tag{51}$$

By solving the linear matrix inequality (11) for each node, $\sigma_i$ and $P_i > 0$ are obtained as $\sigma_1 = \sigma_5 = 0.0021$, $\sigma_2 = \sigma_6 = 1.83 \times 10^{-6}$, $\sigma_3 = \sigma_4 = \sigma_7 = \sigma_8 = \sigma_9 = \sigma_{10} = \sigma_{11} = \sigma_{12} = 0.0024$, and

$$P_1 = 10^3 \times \begin{bmatrix} 1.440 & -0.034 & 0.055 & -0.005 \\ -0.034 & 0.004 & 0.004 & 0 \\ 0.055 & 0.004 & 0.977 & -0.054 \\ -0.005 & 0 & -0.054 & 0.026 \end{bmatrix}, \tag{52}$$

$$P_2 = 10^2 \times \begin{bmatrix} 0.299 & 0 & 0.036 & 0 \\ 0 & 0.696 & 0 & -0.001 \\ 0.036 & 0 & 0.298 & 0 \\ 0 & -0.001 & 0 & 3.999 \end{bmatrix}, \tag{53}$$

$$P_{12} = \begin{bmatrix} 1.44 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1.342 & 0.529 \\ 0 & 0 & 0.529 & 3.496 \end{bmatrix}. \tag{54}$$

Note that $P_1 = P_5$, $P_2 = P_6$ and $P_3 = P_4 = P_7 = P_8 = P_9 = P_{10} = P_{11} = P_{12}$. Under the proposed distributed estimation architecture (9) and (10), nodes are able to closely estimate the process states and inputs as shown in Figure 2 and 3, respectively. ▲

**Example 2.** In this example, we increase the number of active nodes in the sensor network to 8 as depicted in Figure 4. The sensing capability of each agent is the same as in Example 1. Note that, because of the change in the number of active nodes, the design parameters are adjusted accordingly as $\sigma_1 = \sigma_3 = \sigma_5 = \sigma_7 = 0.0021$, $\sigma_2 = \sigma_4 = \sigma_6 = \sigma_8 = 1.83 \times 10^{-6}$, $\sigma_9 = \sigma_{10} = \sigma_{11} = \sigma_{12} = 0.0024$, and $P_1 = P_3 = P_5 = P_7$, $P_2 = P_4 = P_6 = P_8$, $P_9 = P_{10} = P_{11} = P_{12}$, where $P_1$, $P_2$ and $P_{12}$ are the same as (52), (53), and (54), respectively. Other parameters and gains are also kept the same. Figures 5 and 6 show the performance of the sensor network for the proposed distributed estimation architecture. It can be seen that the estimates in this case are slightly better than the ones in Example 1 (i.e, Figure 2 and Figure 3) as a result of increasing the number of active nodes in the sensor network. ▲

**Example 3.** In this example, we consider a sensor network with 8 active nodes and 4 passive nodes as in Example 2 (Figure 4), but change the system output matrices for each node as follows

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{55}$$

$$C_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{56}$$

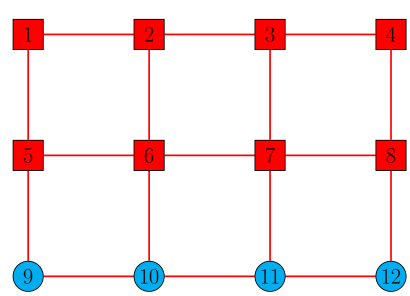


Fig. 4. Communication graph of the sensor network in Example 2 and 3 with 8 active nodes and 4 passive nodes (lines denote communication links, squares denote active nodes, and circles denote passive nodes).

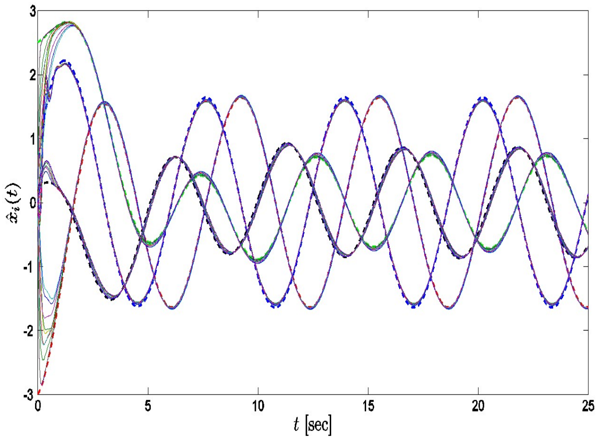


Fig. 5. State estimates of the sensor network in Example 2 with 8 active nodes and 4 passive nodes under the proposed architecture (9) and (10) (the dash lines denote the states of the actual process and the solid lines denote the state estimates of nodes).

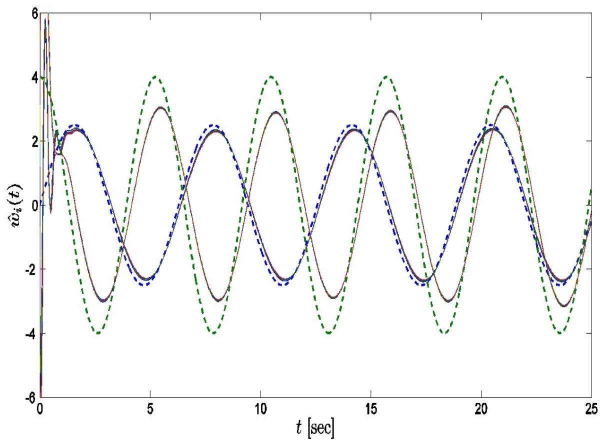


Fig. 6. Input estimates of the sensor network in Example 2 with 8 active nodes and 4 passive nodes under the proposed architecture (9) and (10) (the dash lines denote the inputs of the actual process and the solid lines denote the input estimates of nodes).

$$C_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{57}$$

where $C_1 = C_5 = C_9$, $C_2 = C_4 = C_6 = C_8 = C_{10} = C_{12}$ and $C_3 = C_7 = C_{11}$. Note that for the odd index nodes, the pair $(A, C_i)$ is not observable. We also choose $K_i = \mathrm{diag}([100; 100])$, $\alpha = 50$, and $\gamma = 0.1$.

Here, the observer gain $L_i$ is chosen such that

$$L_1 = \begin{bmatrix} 19.069 & -3.814 \\ -0.486 & 0.097 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \tag{58}$$

$$L_2 = \begin{bmatrix} -2.388 & 0.358 \\ 5.831 & -0.804 \\ 0.428 & -2.398 \\ -1.038 & 6.765 \end{bmatrix}, \tag{59}$$

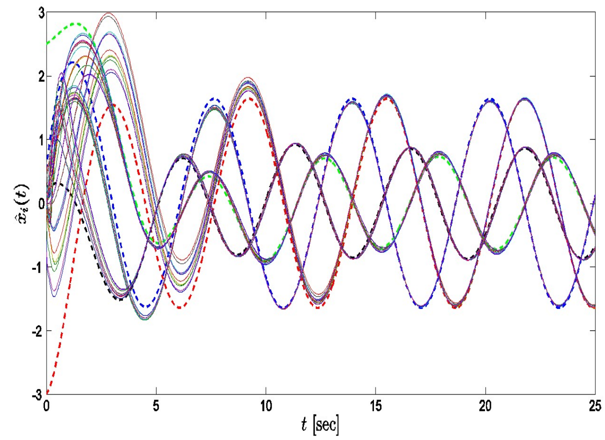


Fig. 7. State estimates of the sensor network in Example 3 with 12 active nodes under the proposed architecture (9) and (10) (the dash lines denote the states of the actual process and the solid lines denote the state estimates of nodes).

$$L_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -3.845 & 19.225 \\ -0.501 & 2.506 \end{bmatrix}, \tag{60}$$

with $L_1 = L_5 = L_9$, $L_2 = L_4 = L_6 = L_8 = L_{10} = L_{12}$ and $L_3 = L_7 = L_{11}$. By solving the linear matrix inequality (11) for each node, $\sigma_i$ and $P_i > 0$ are obtained as $\sigma_1 = \sigma_5 = 0.0024$, $\sigma_2 = \sigma_4 = \sigma_6 = \sigma_8 = 1.83 \times 10^{-6}$, $\sigma_3 = \sigma_7 = 0.002$, $\sigma_9 = \sigma_{10} = \sigma_{11} = \sigma_{12} = 0.0024$, and

$$P_1 = \begin{bmatrix} 1.440 & 0 & 0 & 0 \\ 0 & 1.000 & 0 & 0 \\ 0 & 0 & 697.827 & -23.018 \\ 0 & 0 & -23.018 & 18.832 \end{bmatrix}, \tag{61}$$

$$P_3 = 10^3 \times \begin{bmatrix} 1.478 & -0.034 & 0 & 0 \\ -0.034 & 0.004 & 0 & 0 \\ 0 & 0 & 0.002 & 0 \\ 0 & 0 & 0 & 0.006 \end{bmatrix}, \tag{62}$$

with $P_1 = P_5$, $P_2 = P_4 = P_6 = P_8$, $P_3 = P_7$, and $P_9 = P_{10} = P_{11} = P_{12}$, where $P_2$ and $P_{12}$ are the same as (53) and (54) in Example 1, respectively. Figure 7 and 8 show that under the proposed distributed estimation architecture, nodes are able to closely estimate the process states and inputs, although some active nodes are not able to fully observe the process. Specifically, we can observe that the performances get better over time. ▲

## V. Conclusion

A distributed input and state estimation architecture was investigated for heterogeneous sensor networks having nodes with active and passive information processing roles and nonidentical sensor modalities. It was shown that the proposed framework utilizes local information not only during the execution of the proposed estimation algorithm but also in its design; that is, global stability is guaranteed once each node satisfies given local stability conditions independent from the graph topology and neighboring information of

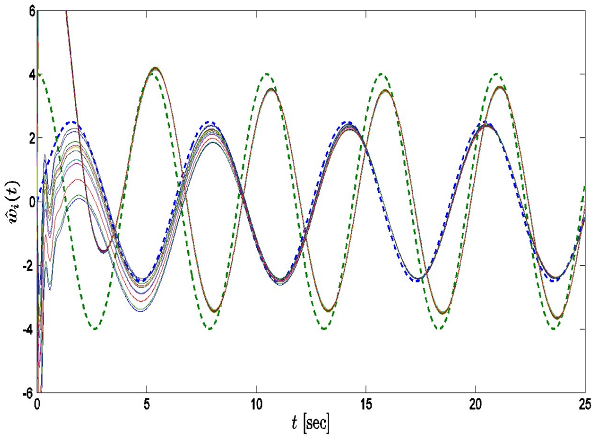


Fig. 8. Input estimates of the sensor network in Example 3 with 12 active nodes under the proposed architecture (9) and (10) (the dash lines denote the inputs of the actual process and the solid lines denote the input estimates of nodes).

these nodes. In addition, passivity or passivity-like assumptions often made in the classical input and state estimation literature were further relaxed utilizing linear matrix inequalities. Several numerical examples illustrated the efficacy of the proposed architecture. Future research will include extensions of the proposed framework to handle sensor networks having nodes with time-varying active and passive information processing roles as well as dynamic data-driven sensor network applications to guide and control autonomous vehicles.

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