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A NEW RESULT ON DISTRIBUTED INPUT AND STATE ESTIMATION FOR HETEROGENEOUS SENSOR NETWORKS*

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ABSTRACT

An important research area in sensor networks is the design and analysis of distributed estimation algorithms for dynamic information fusion in the presence of heterogeneity resulting from ***(i)*** *nonidentical information roles of nodes and* ***(ii)*** *nonidentical modalities of nodes. In particular,* ***(i)*** *implies that both active (i.e., subject to observations of a process of interest) and passive (i.e., subject to no observations) nodes can be present in the sensor network. Furthermore,* ***(ii)*** *implies that active nodes can observe different measurements from a process (e.g., a subset of active nodes can observe position measurements and the rest can observe velocity measurements for a target tracking problem). In this paper, we focus on heterogeneous sensor networks, sensor networks with* ***(i)*** *and* ***(ii)****, and present a new distributed input and state estimation approach. In addition to the presented theoretical contribution including the stability and performance of the proposed estimation approach, an illustrative numerical example is also given to demonstrate its efficacy.*

1 INTRODUCTION

Advances in integrated microsystems open up a broad spectrum of sensor network applications. To this end, an important research area in sensor networks is the design and analysis of distributed estimation algorithms for dynamic information fusion in the presence of heterogeneity resulting from ***(i)*** nonidentical information roles of nodes (e.g., see [1–4]) and ***(ii)*** nonidentical modalities of nodes (e.g., see [5, 6]). In particular, ***(i)*** implies that both active (i.e., subject to observations of a process of interest) and passive (i.e., subject to no observations) nodes can be present in the sensor network. Furthermore, ***(ii)*** implies that active nodes can observe different measurements from a process (e.g., a subset of active nodes can observe position measurements and the rest of active nodes can observe velocity measurements for a target tracking problem).

To address distributed input and state estimation in heterogeneous sensor networks, sensor networks with ***(i)*** and ***(ii)***, the authors recently proposed an algorithm in [7] (also in [8] that expands the results of [7]). The key feature of their algorithm was that it utilizes local information not only during the execution of the distributed input and state estimation law but also in its design (i.e., global stability is guaranteed once each node satisfies given local stability conditions). Yet, it was observed that due to a theoretical conservatism resulting from their approach, the selection of the design parameters of their algorithm may not be always trivial to achieve an acceptable estimation performance (e.g., see Figure 3 in [7]). To address this drawback in this paper,

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we present a new distributed input and state estimation approach. Specifically, by theoretically restructuring the distributed state and input estimation law presented in [7] using a Lyapunov formalism and adding cross coupling terms, it is shown that the new approach of this paper achieves a desired state and input estimation performance with design parameters being easy to tune. Furthermore, the proposed approach still has all the benefits of our former work in [7] (e.g., utilizing local information both during the execution of this approach and in its design). In addition to the presented theoretical contribution including the stability and performance of the proposed estimation approach, an illustrative numerical example is also given to demonstrate its efficacy.

The organization of this paper is as follows. Section 2 introduces mathematical preliminaries for the main results of this paper. In Section 3, we present design and analysis of the new distributed input and state estimation architecture, where the aforementioned illustrative numerical example is included in Section 4. Finally, concluding remarks are summarized in Section 5.

2 MATHEMATICAL PRELIMINARIES

We use a standard notation. Specifically, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrices, $\mathbf{1}_n$ denotes the $n \times 1$ vector of all ones, and I_n denotes the $n \times n$ identity matrix. In addition, we write $(\cdot)^{\mathrm{T}}$ for transpose, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ for the minimum and maximum eigenvalue of the Hermitian matrix A, respectively, $\lambda_i(A)$ for the i-th eigenvalue of A, where A is symmetric and the eigenvalues are ordered from least to greatest value, $\mathrm{diag}(a)$ for the diagonal matrix with the vector a on its diagonal, $[x]_i$ for the entry of the vector x on the i-th row, and A_{ij} for the entry of the matrix A on the i-th row and j-th column.

Next, we recall some basic notions from graph theory and refer to textbooks [9] and [10] for details. Specifically, an undirected graph $\mathcal{G}$ is defined by a set $\mathcal{V}_{\mathcal{G}} = \{1,\ldots,N\}$ of nodes and a set $\mathcal{E}_{\mathcal{G}} \subset \mathcal{V}_{\mathcal{G}} \times \mathcal{V}_{\mathcal{G}}$ of edges. If $(i,j) \in \mathcal{E}_{\mathcal{G}}$, then the nodes i and j are neighbors and the neighboring relation is indicated with $i \sim j$. The degree of a node is given by the number of its neighbors. Letting d_i be the degree of node i, then the degree matrix of a graph $\mathcal{G}$, $\mathcal{D}(\mathcal{G}) \in \mathbb{R}^{N\times N}$, is given by $\mathcal{D}(\mathcal{G}) \triangleq \mathrm{diag}(d)$, $d = [d_1,\ldots,d_N]^{\mathrm{T}}$. A path $i_0 i_1 \ldots i_L$ is a finite sequence of nodes such that $i_{k-1} \sim i_k$, $k = 1,\ldots,L$, and a graph $\mathcal{G}$ is connected if there is a path between any pair of distinct nodes. The adjacency matrix of a graph $\mathcal{G}$, $\mathcal{A}(\mathcal{G}) \in \mathbb{R}^{N\times N}$, is given by $[\mathcal{A}(\mathcal{G})]_{ij} = 1$ if $(i,j) \in \mathcal{E}_{\mathcal{G}}$ and $[\mathcal{A}(\mathcal{G})]_{ij} = 0$ otherwise. The Laplacian matrix of a graph, $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_{+}^{N\times N}$, playing a central role in many graph-theoretic treatments of sensor networks, is given by $\mathcal{L}(\mathcal{G}) \triangleq \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G})$. The spectrum of the Laplacian of an undirected and connected graph can be ordered as $0 = \lambda_1(\mathcal{L}(\mathcal{G})) < \lambda_2(\mathcal{L}(\mathcal{G})) \le \cdots \le \lambda_N(\mathcal{L}(\mathcal{G}))$ with $\mathbf{1}_N$ as the eigenvector corresponding to the zero eigenvalue $\lambda_1(\mathcal{L}(\mathcal{G}))$ and $\mathcal{L}(\mathcal{G})\mathbf{1}_N = \mathbf{0}_N$ and $\mathrm{e}^{\mathcal{L}(\mathcal{G})}\mathbf{1}_N = \mathbf{1}_N$. Here, we assume that the graph $\mathcal{G}$ of a given sensor network is undirected and connected.

3 PROPOSED APPROACH

While we propose a new distributed input and state estimation approach for heterogeneous sensor networks in this paper, we follow the same problem setup outlined in [7]. Specifically, we consider a process with the dynamics given by

$$\dot{x}(t) = Ax(t) + Bw(t), \quad x(0) = x_0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ denotes the process internal state vector, $w(t) \in \mathbb{R}^p$ denotes an unknown bounded input of the process with a bounded time rate of change, $A \in \mathbb{R}^{n\times n}$ denotes the Hurwitz system matrix, and $B \in \mathbb{R}^{n\times p}$ is the system input matrix.

We next consider a sensor network with N nodes exchanging information among each other using their local measurements according to an undirected and connected graph $\mathcal{G}$. Using the terminology of [1–4], a node i, $i = 1,\ldots,N$, is said to be an active node if it is subject to the observation of the process (1) given by

$$y_i = C_i x(t), \tag{2}$$

where $y_i \in \mathbb{R}^p$ and $C_i \in \mathbb{R}^{p\times n}$ denote the measurable process output and the system output matrix for node i, $i = 1,\ldots,N$, respectively. Furthermore, a node i, $i = 1,\ldots,N$, is said to be a passive node when it has no observation of the process (1).

Here, we are interested in the problem of distributively estimating the unmeasurable state $x(t)$ and the unknown input $w(t)$ of the process given by (1) using a sensor network, where active nodes are subject to the observation given by (2). Note that the assumption of A being Hurwitz results from the fact that there are passive nodes in the sensor network; thus, it does not result from the distributed estimation approach proposed in the next section.

We now propose a new input and state estimation law given by

$$\begin{aligned}\dot{\hat{x}}_i(t) &= A\hat{x}_i(t) + B\hat{w}_i(t) + g_i L_i\big(y_i(t) - C_i\hat{x}_i(t)\big) \\ &\quad - \alpha M_i \sum_{j=1}^{N} a_{ij}\big(\hat{x}_i(t) - \hat{x}_j(t)\big) + \alpha S_i \sum_{j=1}^{N} a_{ij}\big(\hat{w}_i(t) - \hat{w}_j(t)\big), \\ &\qquad \hat{x}_i(0) = \hat{x}_{i0},\end{aligned} \tag{3}$$

$$\begin{aligned}\dot{\hat{w}}_i(t) &= g_i K_i\big(y_i(t) - C_i\hat{x}_i(t)\big) - \sigma_i K_i \hat{w}_i(t) \\ &\quad + \alpha T_i \sum_{j=1}^{N} a_{ij}\big(\hat{x}_i(t) - \hat{x}_j(t)\big) - \alpha N_i \sum_{j=1}^{N} a_{ij}\big(\hat{w}_i(t) - \hat{w}_j(t)\big), \\ &\qquad \hat{w}_i(0) = \hat{w}_{i0},\end{aligned} \tag{4}$$

for node i, $i = 1,\dots,N$. In (3) and (4), $\hat{x}(t) \in \mathbb{R}^n$ denotes the local estimate of $x(t)$ for node i and $\hat{w}(t) \in \mathbb{R}^p$ denotes the local input estimate of $w(t)$ for node i. Furthermore, $L_i \in \mathbb{R}^{n\times p}$ and $K_i \in \mathbb{R}^{p\times p}$ are design gain matrices and α and $\sigma_i \in \mathbb{R}$ are positive design coefficients. Finally, $M_i \in \mathbb{R}^{n\times n}$, $S_i \in \mathbb{R}^{n\times p}$, $T_i \in \mathbb{R}^{p\times n}$, and $N_i \in \mathbb{R}^{p\times p}$ are design coefficient matrices. Note that $g_i = 1$ if node i is active and $g_i = 0$ if node i is passive.

Note that we now compare the new distributed input and state estimation law given by (3) and (4) with its counterpart in [7]. For this purpose, the distributed input and state estimation law of [7] has the form

$$\begin{aligned}\dot{\hat{x}}_i(t) = {} & (A-\gamma P_i^{-1})\hat{x}_i(t) + B\hat{w}_i(t) + g_i L_i\big(y_i(t) \\ & -C_i\hat{x}_i(t)\big) - \alpha P_i^{-1}\sum_{i\sim j}(\hat{x}_i(t)-\hat{x}_j(t)), \ \ \hat{x}_i(0)=\hat{x}_{i0}, \quad (5)\\ \dot{\hat{w}}_i(t) = {} & g_iK_i(y_i(t)-C_i\hat{x}_i(t)) - (\sigma_iK_i+\gamma \mathrm{I}_p)\hat{w}_i(t) \\ & -\alpha\sum_{i\sim j}(\hat{w}_i(t)-\hat{w}_j(t)), \ \ \hat{w}_i(0)=\hat{w}_{i0}, \quad (6)\end{aligned}$$

where $P_i > 0$ is a gain satisfying the linear matrix inequality

$$R_i = \begin{bmatrix} \bar{A}_i^{\mathrm{T}}P_i + P_i\bar{A}_i & -P_iB + g_iC_i^{\mathrm{T}}K_i^{\mathrm{T}} \\ -B^{\mathrm{T}}P_i + g_iK_iC_i & -2\sigma_iK_i \end{bmatrix} \le 0, \tag{7}$$

with $\bar{A}_i \triangleq A - g_iL_iC_i$. As noted in [7], the terms "$-\gamma P_i^{-1}\hat{x}_i(t)$" and "$-(\sigma_iK_i+\gamma \mathrm{I}_p)\hat{w}_i(t)$" appearing respectively in (5) and (6) are often referred as leakage terms. To this end, if the gains "γP_i^{-1}" and "$\sigma_iK_i+\gamma \mathrm{I}_p$" multiplying these terms are not small, then they can result in poor performance as well-known. Yet, since "σ_iK_i" also appears in the linear matrix inequality given by (7), this term may not always be selected as small while satisfying (7) either due to the magnitude of the term "$-P_iB+g_iC_i^{\mathrm{T}}K_i$" being not small or a computational conservatism. Thus, (5) and (6) of [7] may not always yield to an acceptable performance.

In contrast to (5) and (6) of [7] discussed in above, the new input and state estimation law given by (3) and (4) only has one leakage term "$-\sigma_iK_i\hat{w}_i(t)$" that appears on the latter equation. Furthermore, as discussed at the end of this section (also see Section 4), the gain "σ_iK_i" of this term can be made sufficiently small here, and hence, the proposed approach of this paper has the capability to achieve better estimation performance as compared with our recent results documented in [7].

Next, let

$$\tilde{x}_i(t) \triangleq x(t) - \hat{x}_i(t) \in \mathbb{R}^n, \tag{8}$$

$$\tilde{w}_i(t) \triangleq \hat{w}_i(t) - w(t) \in \mathbb{R}^p. \tag{9}$$

Taking the time derivative of (8), one can write

$$\begin{aligned}\dot{\tilde{x}}_i(t) &= \dot{x}(t) - \dot{\hat{x}}_i(t) \\ &= Ax(t) + Bw(t) - A\hat{x}_i(t) - B\hat{w}_i(t) - g_iL_i\big(y_i(t) - C_i\hat{x}_i(t)\big) \\ &\quad +\alpha M_i\sum_{j=1}^N a_{ij}\big(\hat{x}_i(t)-\hat{x}_j(t)\big) - \alpha S_i\sum_{j=1}^N a_{ij}\big(\hat{w}_i(t)-\hat{w}_j(t)\big) \\ &= A\tilde{x}_i(t) - B\tilde{w}_i(t) - g_iL_iC_i\tilde{x}_i(t) \\ &\quad -\alpha M_i\sum_{j=1}^N a_{ij}\big(\tilde{x}_i(t)-\tilde{x}_j(t)\big) - \alpha S_i\sum_{j=1}^N a_{ij}\big(\tilde{w}_i(t)-\tilde{w}_j(t)\big) \\ &= (A-g_iL_iC_i)\tilde{x}_i(t) - B\tilde{w}_i(t) - \alpha M_i\sum_{j=1}^N a_{ij}\big(\tilde{x}_i(t)-\tilde{x}_j(t)\big) \\ &\quad -\alpha S_i\sum_{j=1}^N a_{ij}\big(\tilde{w}_i(t)-\tilde{w}_j(t)\big) \\ &= (A-g_iL_iC_i)\tilde{x}_i(t) - B\tilde{w}_i(t) \\ &\qquad -\alpha M_i\sum_{j=1}^N \mathcal{L}_{ij}\tilde{x}_j(t) - \alpha S_i\sum_{j=1}^N \mathcal{L}_{ij}\tilde{w}_j(t). \end{aligned} \tag{10}$$

where $\mathcal{L}_{ij}$ is the entry of the Laplacian matrix on the i-th row and j-th column. Furthermore, the time derivative of (9) can be written as

$$\begin{aligned}\dot{\tilde{w}}_i(t) &= g_iK_iC_i\big(x(t)-\hat{x}_i(t)\big) - \sigma_iK_i\big(\tilde{w}_i(t) \\ &\quad +w(t)\big) + \alpha T_i\sum_{j=1}^N a_{ij}\big(x(t)-\tilde{x}_i(t)-x(t)+\tilde{x}_j(t)\big) \\ &\quad -\alpha N_i\sum_{j=1}^N a_{ij}\big(\tilde{w}_i(t)+w(t)-\tilde{w}_j(t)-w(t)\big) - \dot{w}(t) \\ &= g_iK_iC_i\tilde{x}_i(t) - \sigma_iK_i\big(\tilde{w}_i(t)+w(t)\big) - \dot{w}(t) \\ &\quad -\alpha T_i\sum_{j=1}^N a_{ij}\big(\tilde{x}_i(t)-\tilde{x}_j(t)\big) - \alpha N_i\sum_{j=1}^N a_{ij}\big(\tilde{w}_i(t)-\tilde{w}_j(t)\big) \\ &= g_iK_iC_i\tilde{x}_i(t) - \sigma_iK_i\tilde{w}_i(t) \\ &\quad -\alpha T_i\sum_{j=1}^N \mathcal{L}_{ij}\tilde{x}_j(t) - \alpha N_i\sum_{j=1}^N \mathcal{L}_{ij}\tilde{w}_j(t) - \sigma_iK_iw(t) - \dot{w}(t). \end{aligned} \tag{11}$$

Furthermore, let $z_i = [\tilde{x}_i^{\mathrm{T}}(t), \tilde{w}_i^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{n+p}$. Now, (10) and (11) can be written in a compact form as

$$\begin{aligned}\dot{z}_i(t) = {} & \begin{bmatrix} A-g_iL_iC_i & -B \\ g_iK_iC_i & -\sigma_iK_i \end{bmatrix} z_i(t) - \alpha\sum_{j=1}^N \mathcal{L}_{ij}\begin{bmatrix} M_i & S_i \\ T_i & N_i \end{bmatrix} z_j(t) \\ & + \begin{bmatrix} 0 \\ -\sigma_iK_iw(t) - \dot{w}(t) \end{bmatrix}, \end{aligned} \tag{12}$$

or equivalently,

$$\dot{z}_i(t) = \bar{A}_i z_i(t) - \alpha \sum_{j=1}^{N} \mathcal{L}_{ij} H_i z_j(t) + \phi_i(t), \tag{13}$$

where $\bar{A}_i = \begin{bmatrix} A - g_i L_i C_i & -B \\ g_i K_i C_i & -\sigma_i K_i \end{bmatrix} \in \mathbb{R}^{(n+p)\times(n+p)}$, $H_i = \begin{bmatrix} M_i & S_i \\ T_i & N_i \end{bmatrix} \in \mathbb{R}^{(n+p)\times(n+p)}$ and $\phi_i(t) = \begin{bmatrix} 0 \\ -\sigma_i K_i w(t) - \dot{w}(t) \end{bmatrix} \in \mathbb{R}^{n+p}$. It is of practical importance to note here that one can always choose the design terms L_i, K_i, and σ_i such that $\bar{A}_i$ is Hurwitz, and hence, $\bar{A}_i$ being Hurwitz is assumed for the following results. Here, note also that there exists a unique positive-definite matrix P_i such that

$$\bar{A}_i^{\mathrm{T}} P_i + P_i \bar{A}_i + Q_i = 0, \tag{14}$$

holds for a given positive-definite matrix Q_i.

Now, let the aggregated vector be given by $z(t) \triangleq [z_1^{\mathrm{T}}(t), z_2^{\mathrm{T}}(t), \ldots, z_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{(n+p)N}$. To this end, (13) can be written in a compact form as

$$\begin{aligned} \dot{z}(t) &= \begin{bmatrix} \bar{A}_1 & & 0 \\ & \ddots & \\ 0 & & \bar{A}_N \end{bmatrix} z(t) \\ &\quad - \alpha \begin{bmatrix} \mathcal{L}_{11}H_1 & \mathcal{L}_{12}H_1 & \ldots & \mathcal{L}_{1N}H_1 \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{L}_{N1}H_N & \mathcal{L}_{N2}H_N & \ldots & \mathcal{L}_{NN}H_N \end{bmatrix} z(t) + \begin{bmatrix} \phi_1(t) \\ \vdots \\ \phi_N(t) \end{bmatrix} \\ &= \begin{bmatrix} \bar{A}_1 & & 0 \\ & \ddots & \\ 0 & & \bar{A}_N \end{bmatrix} z(t) - \alpha \begin{bmatrix} H_1 & & 0 \\ & \ddots & \\ 0 & & H_N \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} \mathcal{L}_{11}\mathrm{I}_{n+p} & \mathcal{L}_{12}\mathrm{I}_{n+p} & \ldots & \mathcal{L}_{1N}\mathrm{I}_{n+p} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{L}_{N1}\mathrm{I}_{n+p} & \mathcal{L}_{N2}\mathrm{I}_{n+p} & \ldots & \mathcal{L}_{NN}\mathrm{I}_{n+p} \end{bmatrix} z(t) + \phi(t) \\ &= \bar{A}z(t) - \alpha H(\mathcal{L}(\mathcal{G}) \otimes \mathrm{I}_{n+p}) z(t) + \phi(t). \end{aligned} \tag{15}$$

where $\mathcal{L}(\mathcal{G})$ is the Laplacian matrix. Considering the process given by (1) and the distributed input and state estimation architecture given by (3) and (4); it can be shown that if the matrix H_i is selected as $H_i = P_i^{-1}$ and nodes exchange information using local measurement subject to an undirected and connected graph $\mathcal{G}$, then the error dynamics given by (15) is uniformly bounded. The proof of this result will be reported elsewhere, but for interested readers, it follows by utilizing the Lyapunov function candidate given by $V\big(z(t)\big) = z^{\mathrm{T}}(t) P z(t)$. Note that one can also show $\|z(t)\|_2^2 \leq \sqrt{\frac{\lambda_{\max}(P))\mu^2}{\lambda_{\min}(P))}}$ for $t \geq T$, where $\mu \triangleq \frac{2\|P\|_2 \bar{\phi}}{\lambda_{\min}(\bar{Q})}$ with $\|\phi\|_2 \leq \bar{\phi}$ and $\bar{Q} = Q + 2\alpha(\mathcal{L}(\mathcal{G}) \otimes \mathrm{I}_{n+p})$.

Since the ultimate bound given in the last part of the above paragraph depends on the design parameters of the proposed distributed input and state estimation architecture, it can be used as design metric such that the design parameters can be judiciously selected to make the above ultimate bound small. For example, we may choose a small value for σ_i and K_i such that the bound $\bar{\phi}$ becomes small, which appears on the above ultimate bound expression through the term μ. Note that in [7] we did not always have a great flexibility in choosing σ_i and K_i small, since this may make the linear matrix inequality (7) infeasible.

4 ILLUSTRATIVE EXAMPLE

In this section, we illustrate the results presented in the previous section. For this purpose, we consider the benchmark process in [7] that is composed of two decoupled systems with the dynamics given by (1), where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_{\mathrm{n}1}^2 & -2\omega_{\mathrm{n}1}\xi_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_{\mathrm{n}2}^2 & -2\omega_{\mathrm{n}2}\xi_2 \end{bmatrix}, \tag{16}$$

$$B = \begin{bmatrix} 0 & 0 \\ \omega_{\mathrm{n}1}^2 & 0 \\ 0 & 0 \\ 0 & \omega_{\mathrm{n}2}^2 \end{bmatrix}, \tag{17}$$

with $\omega_{n1} = 1.2, \xi_1 = 0.9, \omega_{n2} = 1.3$, and $\xi_2 = 0.5$. This process can represent a linearized simple vehicle model with the first and third states corresponding to the positions in the x and y directions, respectively, while the second and fourth states corre-

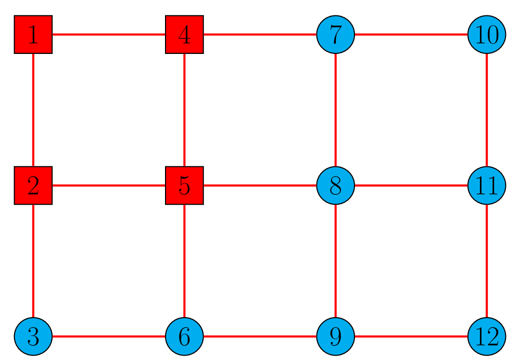

FIGURE 4.1. Communication graph of the sensor network with four active nodes 1, 2, 4, 5 and eight passive nodes 3, 6, 7, 8, 9, 10, 11, 12 (lines denote communication links, squares denote active nodes, and circles denote passive nodes).

sponding to the velocities in the x and y directions, respectively. The initial conditions are set to $x_0^{\mathrm{T}} = [-3,\ 0.5,\ 2.5,\ 0.25]^{\mathrm{T}}$. In addition, we consider the input given by

$$w(t) = \begin{bmatrix} 2.5\sin(0.3t) \\ 0.5\cos(0.5t) \end{bmatrix}. \tag{18}$$

For the numerical results presented in this section, we consider a sensor network with 12 nodes exchanging information over an undirected and connected graph topology, where there are 4 active nodes and 8 passive nodes as shown in Figure 4.1. Each node's sensing capability is represented by (2) with the output matrices

$$C_i = \begin{bmatrix} 1\ 0\ 0\ 0 \\ 0\ 0\ 1\ 0 \end{bmatrix}, \tag{19}$$

for the odd index nodes and

$$C_i = \begin{bmatrix} 0\ 1\ 0\ 0 \\ 0\ 0\ 0\ 1 \end{bmatrix}. \tag{20}$$

for the even index nodes. Moreover, all nodes are subject to zero initial conditions and we set $K_i = \mathrm{diag}([10;10])$ and $\sigma_i = 0.00001$ for $i = 1,\ldots,N$. For the observer gain L_i, the odd index nodes are subject to

$$L_i = \begin{bmatrix} 8.137 & 0 \\ 8.102 & 0 \\ 0 & 8.538 \\ 0 & 11.446 \end{bmatrix}, \tag{21}$$

while the even index nodes are subject to

$$L_i = \begin{bmatrix} -13.389 & 1.384 \\ 19.178 & -1.960 \\ 1.510 & -13.402 \\ -2.153 & 20.143 \end{bmatrix}. \tag{22}$$

For active and passive nodes we set $\alpha = 25$. In addition, we obtain from (14)

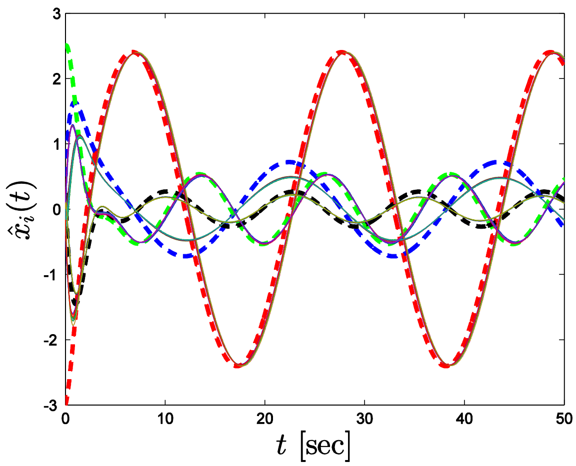

FIGURE 4.2. State estimates of the sensor network with four active nodes and eight passive nodes under the proposed architecture (3) and (4) (the dash lines denote the states of the actual process and the solid lines denote the state estimates of nodes).

$$P_1 = \begin{bmatrix} 12.42 & -1.40 & -2.99 & -0.71 & 7.26 & -3.13 \\ -1.40 & 6.53 & -0.16 & -0.80 & 4.28 & -0.70 \\ -2.99 & -0.16 & 5.17 & -3.18 & -2.49 & -0.99 \\ -0.71 & -0.80 & -3.18 & 3.55 & -0.70 & 2.54 \\ 7.26 & 4.28 & -2.49 & -0.70 & 10.55 & -3.15 \\ -3.13 & -0.70 & -0.99 & 2.54 & -3.15 & 3.06 \end{bmatrix}, \tag{23}$$

$$P_2 = \begin{bmatrix} 7.84 & 5.45 & -0.54 & -0.26 & 1.97 & 0.13 \\ 5.45 & 7.06 & -0.28 & -0.07 & 5.45 & -0.28 \\ -0.54 & -0.28 & 5.82 & 4.21 & 0.04 & 1.92 \\ -0.26 & -0.07 & 4.21 & 5.52 & -0.26 & 4.20 \\ 1.97 & 5.45 & 0.04 & -0.26 & 10.49 & -1.51 \\ 0.13 & -0.28 & 1.92 & 4.20 & -1.51 & 7.62 \end{bmatrix}, \tag{24}$$

$$P_3 = \begin{bmatrix} 0.21 & 0.05 & 0 & 0 & 0.21 & 0 \\ 0.05 & 0.11 & 0 & 0 & 0.05 & 0 \\ 0 & 0 & 0.24 & 0.05 & 0 & 0.24 \\ 0 & 0 & 0.05 & 0.15 & 0 & 0.05 \\ 0.21 & 0.05 & 0 & 0 & 0.41 & 0 \\ 0 & 0 & 0.24 & 0.05 & 0 & 0.42 \end{bmatrix}. \tag{25}$$

Note that $P_1 = P_5$, $P_2 = P_4$ and $P_3 = P_6 = P_7 = P_8 = P_9 = P_{10} = P_{11} = P_{12}$. Based on the matrix P_i, $i = 1,2,\ldots,12$, we obtain $H_i = \begin{bmatrix} M_i & S_i \\ T_i & N_i \end{bmatrix} = P_i^{-1}$ and matrices M_i, S_i, T_i, and N_i are selected accordingly.

Under the proposed distributed estimation architecture (3)

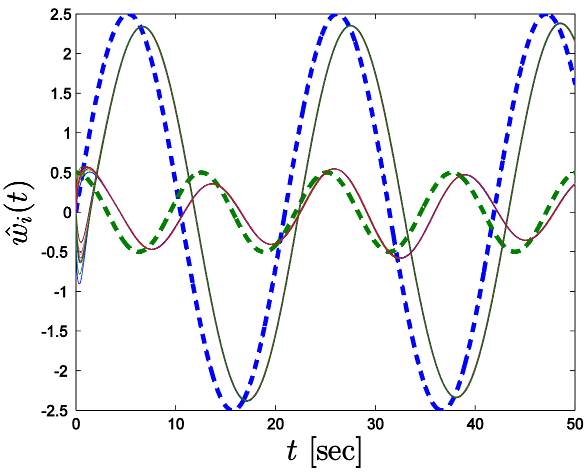

FIGURE 4.3. Input estimates of the sensor network with four active nodes and eight passive nodes under the proposed architecture (3) and (4) (the dash lines denote the inputs of the actual process and the solid lines denote the input estimates of nodes).

and (4), nodes are able to closely estimate the process states and inputs as shown in Figures 4.2 and 4.3, respectively. Recall that in [7], although the state estimation is good (see Figure 2 in [7]), the input estimation performance depends on the number of active nodes in the sensor networks. When passive nodes dominate the sensor networks, the input estimation cannot give a desired estimation performance (see Figure 3 in [7]). The proposed algorithm in this paper; however, can closely estimate process input with only a small subset of active nodes as shown in Figure 4.3.

5 CONCLUSION

In order to contribute to the previous studies in heterogeneous sensor networks, we proposed a new distributed input and state estimation approach. In addition, the stability of the overall sensor network subject to the proposed approach as well as its performance were analyzed in detail. The illustrative example had shown that nodes can closely estimate both states and inputs of the process, and hence, validated the proposed theoretical contribution of this paper. Our future research will include generalizing the current results to the case where the active-passive role of each agent varies over time and considering the noise in the sensors as stochastic processes.

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