Distributed Coestimation in Heterogeneous Sensor Networks with Time-Varying Active and Passive Node Roles[⋆]

Dzung Tran[†], Tansel Yucelen[‡], Sarangapani Jagannathan[◇], and David Casbeer[*]

***Abstract*— In this paper, we address the problem of system-theoretic dynamic information fusion in time-varying heterogeneous sensor networks. This class of sensor networks involves nodes that receive observations from a process of interest (active nodes) and nodes that do not receive any information (passive nodes), where the active and passive node roles can be varying with respect to time. At any given time, in addition, active nodes are allowed to have nonidentical modalities such that they can observe different measurements from the process. Specifically, we propose a new *distributed input and state "coestimation"* architecture for time-varying heterogeneous sensor networks, where time evolution of input and state updates of each node *both* depend on the local input and state information exchanges. The stability and performance of the overall sensor network are guaranteed once the *local sufficient stability conditions* for each node are satisfied. As compared with our recent *distributed input and state "estimation"* approach for the same problem, where time evolution of input (respectively, state) update of each node *only* depends on the local input (respectively, state) information exchange, our illustrative numerical example also demonstrates a substantially improved dynamic input and state fusion performance.**

I. Introduction

One of the fundamental problems in distributed algorithm synthesis and analysis in sensor networks for dynamic information fusion, which is of paramount importance to their practical applications, is *heterogeneity*. Specifically, these networks often involve nodes with *heterogeneous information roles*; that is, active nodes and passive nodes. While the authors of [1]–[7] present notable contributions to sensor networks with (time-invariant and/or time-varying) active and passive node roles, their results are applicable to practical scenarios only when each node obey scalar integrator dynamics. In addition to heterogeneous information roles, active nodes at any given time can also involve *heterogeneous modalities* such that they can observe different measurements from the process. To this end, the authors of [8]–[10] consider nonidentical modalities of sensor nodes in their distributed algorithms. In particular, the result documented in [8] ignores the possibility of having passive nodes in the network since it requires all nodes to be active in the sense of receiving observations from a process of interest. Although the authors of [9] implicitly consider nodes that can have time-invariant active and passive information roles, their analysis involve *global sufficient stability conditions*, which can be impractical for sensor networks having sufficiently large set of nodes. Recently, our result documented in [10] considers nodes that can have either time-invariant or time-varying active and passive information roles with nodes having nonidentical modalities under *local sufficient stability conditions* for each node (see also [11] for a preliminary version of this result). Yet, as it is highlighted in [Section 4.3, 10], tuning the resulting distributed algorithm for a satisfactory performance can be a challenge in the presence of time-varying set of active and passive nodes.

In this paper, we address the problem of system-theoretic dynamic information fusion in *time-varying heterogeneous sensor networks* — sensor networks with time-varying set of active and passive information roles subject to nonidentical modalities. Specifically, we propose a new distributed architecture entitled *distributed input and state "coestimation"*, where time evolution of input and state updates of each node *both* depend on the local input and state information exchanges. The stability and performance of the overall sensor network are guaranteed once the *local sufficient stability conditions* for each node are satisfied. As compared with our recent *distributed input and state "estimation"* approach documented in [10] for the same problem, where time evolution of input (respectively, state) update of each node *only* depends on the local input (respectively, state) information exchange, our illustrative numerical example also demonstrates a substantially improved dynamic input and state fusion performance. Finally, a preliminary version of this paper appeared in [12], which only considers distributed algorithm synthesis and analysis for *time-invariant* heterogeneous sensor networks.

The organization of this paper is as follows. In Section II, we introduce necessary notations and definitions for the main results of this paper. We then present synthesis and analysis of the proposed distributed input and state coestimation architecture in Section III, where the aforementioned illustrative numerical example involving a comparison with the approach documented in [10] is included in Section IV. Finally, concluding remarks with regard to the results of this

[†]D. Tran is a Graduate Research Assistant of the Mechanical Engineering Department and a member of the Laboratory for Autonomy, Control, Information, and Systems (LACIS, http://lacis.eng.usf.edu/) at the University of South Florida, Tampa, Florida 33620, United States of America (email: dtran3@mail.usf.edu).

[‡]T. Yucelen is an Assistant Professor of the Mechanical Engineering Department and the Director of the Laboratory for Autonomy, Control, Information, and Systems (LACIS, /http://lacis.eng.usf.edu/) at the University of South Florida, Tampa, Florida 33620, United States of America (email: yucelen@usf.edu).

[◇]S. Jagannathan is a Professor of the Electrical and Computer Engineering Department and the Director of the Embedded Control Systems and Networking Laboratory (ECSNL, http://web.mst.edu/∼sarangap/) at the Missouri University of Science and Technology, Rolla, Missouri 65409, United States of America (email: sarangap@mst.edu).

[*]D. Casbeer is affiliated with the Autonomous Control Branch at the Air Force Research Laboratory, Wright-Patterson Air Force Base, Ohio 45433, United States of America (email: david.casbeer@us.af.mil).

[⋆]This research was supported by the Dynamic Data-Driven Applications Systems Program of the Air Force Office of Scientific Research (21051138).

paper are summarized in Section V.

II. Notation and Definitions

Throughout this paper, we use a standard mathematical notation. In particular, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrices, $\mathbf{1}_n$ denotes the $n \times 1$ vector of all ones, and I_n denotes the $n \times n$ identity matrix. Furthermore, we write $(\cdot)^{\mathrm{T}}$ for transpose, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ respectively for the minimum and maximum eigenvalue of the Hermitian matrix A, $\lambda_i(A)$ for the *i*-th eigenvalue of A, where A is symmetric and the eigenvalues are ordered from least to greatest value, $\mathrm{diag}(a)$ for the diagonal matrix with the vector a on its diagonal, $[x]_i$ for the entry of the vector x on the *i*-th row, and A_{ij} for the entry of the matrix A on the *i*-th row and *j*-th column.

We next recall some basic notions from graph theory and refer to excellent textbooks [13] and [14] for details. Specifically, an undirected graph $\mathcal{G}$ is defined by a set $\mathcal{V}_{\mathcal{G}} = \{1, \ldots, N\}$ of nodes and a set $\mathcal{E}_{\mathcal{G}} \subset \mathcal{V}_{\mathcal{G}} \times \mathcal{V}_{\mathcal{G}}$ of edges. If $(i, j) \in \mathcal{E}_{\mathcal{G}}$, then the nodes i and j are neighbors and the neighboring relation is indicated with $i \sim j$. The degree of a node is given by the number of its neighbors. Letting d_i be the degree of node i, then the degree matrix of a graph $\mathcal{G}$, $\mathcal{D}(\mathcal{G}) \in \mathbb{R}^{N\times N}$, is defined by $\mathcal{D}(\mathcal{G}) \triangleq \mathrm{diag}(d)$, $d = [d_1, \ldots, d_N]^{\mathrm{T}}$. A path $i_0 i_1 \ldots i_L$ is a finite sequence of nodes such that $i_{k-1} \sim i_k$, $k = 1, \ldots, L$, and a graph $\mathcal{G}$ is connected if there is a path between any pair of distinct nodes. The adjacency matrix of a graph $\mathcal{G}$, $\mathcal{A}(\mathcal{G}) \in \mathbb{R}^{N\times N}$, is defined by $[\mathcal{A}(\mathcal{G})]_{ij} = 1$ if $(i, j) \in \mathcal{E}_{\mathcal{G}}$ and $[\mathcal{A}(\mathcal{G})]_{ij} = 0$ otherwise. The Laplacian matrix of a graph, $\mathcal{L}(\mathcal{G}) \in \overline{\mathbb{S}}_+^{N\times N}$, playing a central role in many graph-theoretic sensor network treatments, is defined by $\mathcal{L}(\mathcal{G}) \triangleq \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G})$. The spectrum of the Laplacian of an undirected and connected graph can be ordered as $0 = \lambda_1(\mathcal{L}(\mathcal{G})) < \lambda_2(\mathcal{L}(\mathcal{G})) \leq \cdots \leq \lambda_N(\mathcal{L}(\mathcal{G}))$ with $\mathbf{1}_N$ as the eigenvector corresponding to the zero eigenvalue $\lambda_1(\mathcal{L}(\mathcal{G}))$ and $\mathcal{L}(\mathcal{G})\mathbf{1}_N = \mathbf{0}_N$ and $\mathrm{e}^{\mathcal{L}(\mathcal{G})}\mathbf{1}_N = \mathbf{1}_N$. In this paper, it is considered that the graph $\mathcal{G}$ of a given sensor network is undirected and connected.

III. Distributed Input and State Coestimation

Consider a process of interest satisfying the dynamics of the form given by

$$\dot{x}(t) = Ax(t) + Bw(t). \quad x(0) = x_0. \tag{1}$$

In (1), $x(t) \in \mathbb{R}^n$ is an unmeasurable process state vector, $w(t) \in \mathbb{R}^p$ is an unknown bounded input of the process with a bounded time rate of change, $A \in \mathbb{R}^{n\times n}$ is a Hurwitz system matrix, and $B \in \mathbb{R}^{n\times p}$ is the system input matrix. Furthermore, consider a sensor network with N nodes exchanging information among each other using their local measurements according to an undirected and connected graph $\mathcal{G}$. As discussed, we call a node i to be *active* if it receives an observation of the form given by

$$y_i(t) = C_i x(t), \tag{2}$$

from the process (1), where $y_i(t) \in \mathbb{R}^m$ and $C_i \in \mathbb{R}^{m\times n}$ respectively denote a process output and its system output matrix. Likewise, we call a node i to be *passive* if it does not receive any observation from the process (1). Notice from (2) that each node can have nonidentical sensing modalities.

In this paper, we focus on the problem of distributively observing the unmeasurable state $x(t)$ and the unknown input $w(t)$ of the process given by (1) using a sensor network having time-varying active and passive node roles through dynamic information fusion. Specifically, if a node in the heterogeneous sensor network is active for some time instant, then it is subject to the observations of the process given by (2) on that time instant, otherwise it is a passive node and has no observation. Similar to the case in Figure 2d of [15] and without loss of much practical generality, we assume that a node can smoothly change back and forth between active and passive information roles, where the smooth function $g_i(t) \in [0, 1]$ captures such role changes. While each node can have nonidentical sensing modalities as mentioned above, we also assume for the well-posedness of the considered problem here that each active node has complementary properties distributed over the sensor network to guarantee collective observability, even though the pairs (A, C_i), $i = 1, \ldots, N$, may not be locally observable. Mathematically speaking, collective observability is defined as the pair (A, C) is observable, where $C = [C_1^{\mathrm{T}}, C_2^{\mathrm{T}}, \ldots, C_N^{\mathrm{T}}]^{\mathrm{T}}$ (see, for example, [8]–[10]). Hence, a sensor network design is a-priori required to guarantee collective observability at any time in the presence of time-varying active and passive information roles of nodes.

For each node i, $i = 1, \ldots, N$, we now propose the distributed input and state coestimation algorithm given by

$$\begin{aligned}\dot{\hat{x}}_i(t) = {} & A\hat{x}_i(t) + B\hat{w}_i(t) + g_i(t)L_i\big(y_i(t) - C_i\hat{x}_i(t)\big) \\ & -\alpha M_i \sum_{j=1}^{N} a_{ij}\big(\hat{x}_i(t) - \hat{x}_j(t)\big) \\ & +\alpha S_i \sum_{j=1}^{N} a_{ij}\big(\hat{w}_i(t) - \hat{w}_j(t)\big), \quad \hat{x}_i(0) = \hat{x}_{i0},\end{aligned} \tag{3}$$

$$\begin{aligned}\dot{\hat{w}}_i(t) = {} & g_i(t)J_i\big(y_i(t) - C_i\hat{x}_i(t)\big) - \sigma_i K_i \hat{w}_i(t) \\ & +\alpha T_i \sum_{j=1}^{N} a_{ij}\big(\hat{x}_i(t) - \hat{x}_j(t)\big) \\ & -\alpha N_i \sum_{j=1}^{N} a_{ij}\big(\hat{w}_i(t) - \hat{w}_j(t)\big), \quad \hat{w}_i(0) = \hat{w}_{i0},\end{aligned} \tag{4}$$

where $\hat{x}_i(t) \in \mathbb{R}^n$ is the local estimate of $x(t)$ for node i, $\hat{w}_i(t) \in \mathbb{R}^p$ is the local input estimate of $w(t)$ for node i, $L_i \in \mathbb{R}^{n\times p}$, $J_i \in \mathbb{R}^{p\times m}$ and $K_i \in \mathbb{R}^{p\times p}$ are design gain matrices, $M_i \in \mathbb{R}^{n\times n}$, $S_i \in \mathbb{R}^{n\times p}$, $T_i \in \mathbb{R}^{p\times n}$, and $N_i \in \mathbb{R}^{p\times p}$ are additional design gain matrices, and $\alpha \in \mathbb{R}_+$ and $\sigma_i \in \mathbb{R}_+$ are design coefficients. As discussed above, here $g_i(t) \in [0, 1]$ represents a smooth function for each node i, $i = 1, \ldots, N$, which determines whether a node is active or passive at a given time.

Since the time evolution of input and state updates given by (3) and (4) both depend on the local input and state information exchanges (i.e., the coupling terms "$\hat{x}_i(t) - \hat{x}_j(t)$" and "$\hat{w}_i(t) - \hat{w}_j(t)$" that appear both in these updates through a given graph), we use the word "*coestimation*" when referring to the proposed distributed algorithm. As compared with our recent distributed input and state "*estimation*" approach documented in [10], this is the main difference of the proposed distributed coestimation algorithm given by (3) and (4), where the time evolution of input (respectively, state) update of each node only depends the local input (respectively, state) information exchange with the distributed algorithm presented in [10].

We now discuss the stability and performance aspects of the proposed distributed coestimation algorithm presented above. For this purpose, we first define the state and input estimation error vectors respectively as

$$\tilde{x}_i(t) \triangleq x(t) - \hat{x}_i(t), \tag{5}$$
$$\tilde{w}_i(t) \triangleq \hat{w}_i(t) - w(t). \tag{6}$$

Specifically, by taking the derivative of (5) with respect to time, one can write

$$\begin{aligned}\dot{\tilde{x}}_i(t) &= Ax(t) + Bw(t) - A\hat{x}_i(t) - B\hat{w}_i(t) \\ &\quad -g_i(t)L_i\big(y_i(t) - C_i\hat{x}_i(t)\big) \\ &\quad +\alpha M_i \sum_{j=1}^{N} a_{ij}\big(\hat{x}_i(t) - \hat{x}_j(t)\big) \\ &\quad -\alpha S_i \sum_{j=1}^{N} a_{ij}\big(\hat{w}_i(t) - \hat{w}_j(t)\big) \\ &= (A - g_i(t)L_iC_i)\tilde{x}_i(t) - B\tilde{w}_i(t) \\ &\quad -\alpha M_i \sum_{j=1}^{N} \mathcal{L}_{ij}\tilde{x}_j(t) - \alpha S_i \sum_{j=1}^{N} \mathcal{L}_{ij}\tilde{w}_j(t). \end{aligned} \tag{7}$$

where $\mathcal{L}_{ij}$ is the entry of the Laplacian matrix on the *i*-th row and *j*-th column. Furthermore, by taking the derivative of (6) with respect to time, one can also write

$$\begin{aligned}\dot{\tilde{w}}(t) &= g_i(t)J_iC_i\big(x_i(t) - \hat{x}_i(t)\big) - \sigma_iK_i\big(\tilde{w}_i(t) + w(t)\big) \\ &\quad +\alpha T_i \sum_{j=1}^{N} a_{ij}\big(x(t) - \tilde{x}_i(t) - x(t) + \tilde{x}_j(t)\big) \\ &\quad -\alpha N_i \sum_{j=1}^{N} a_{ij}\big(\tilde{w}_i(t) + w(t) - \tilde{w}_j(t) - w(t)\big) - \dot{w}(t) \\ &= g_i(t)J_iC_i\tilde{x}_i(t) - \sigma_iK_i\tilde{w}_i(t) - \alpha T_i \sum_{j=1}^{N} \mathcal{L}_{ij}\tilde{x}_j(t) \\ &\quad -\alpha N_i \sum_{j=1}^{N} \mathcal{L}_{ij}\tilde{w}_j(t) - \sigma_iK_iw(t) - \dot{w}(t). \end{aligned} \tag{8}$$

Next, let $z_i(t) \triangleq [\tilde{x}_i^{\mathrm{T}}(t), \tilde{w}_i^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{n+p}$ be the aggregated error vector; therefore, we can now write (7) and (8) in the compact form

$$\begin{aligned}\dot{z}_i(t) &= \underbrace{\begin{bmatrix} A - g_i(t)L_iC_i & -B \\ g_i(t)J_iC_i & -\sigma_iK_i \end{bmatrix}}_{\bar{A}_i(t)} z_i(t) \\ &\quad -\alpha \sum_{j=1}^{N} \mathcal{L}_{ij} \underbrace{\begin{bmatrix} M_i & S_i \\ T_i & N_i \end{bmatrix}}_{H_i} z_j(t) + \underbrace{\begin{bmatrix} 0 \\ -\sigma_iK_iw(t) - \dot{w}(t) \end{bmatrix}}_{\phi_i(t)} \\ &= \bar{A}_i(t)z_i(t) - \alpha \sum_{j=1}^{N} \mathcal{L}_{ij}H_iz_j(t) + \phi_i(t). \end{aligned} \tag{9}$$

Here, the matrix $\bar{A}_i(t)$ can be rewritten as

$$\begin{aligned}\bar{A}_i(t) = \bar{A}_i\big(g_i(t)\big) &= \underbrace{\begin{bmatrix} A & -B \\ 0 & -\sigma_iK_i \end{bmatrix}}_{\bar{A}_{i,0}} + g_i(t) \underbrace{\begin{bmatrix} -L_iC_i & 0 \\ J_iC_i & 0 \end{bmatrix}}_{\tilde{A}_i}, \\ &= \bar{A}_{i,0} + g_i(t)\tilde{A}_i, \end{aligned} \tag{10}$$

where $g_i(t) \in [0, 1]$. Note that $\bar{A}_{i,0}$ and $\bar{A}_{i,1}$ are the matrices corresponding to $\bar{A}_i(t)$ at $g_i(t) = 0$ and $g_i(t) = 1$, respectively. Hence, $\tilde{A}_i = \bar{A}_{i,1} - \bar{A}_{i,0}$. The following result is now needed:

If there exists a common positive-definite matrix P_i for node i, $i = 1, \ldots, N$, satisfying

$$\bar{A}_{i,0}^{\mathrm{T}}P_i + P_i\bar{A}_{i,0} \leq -\epsilon \mathrm{I}_{n+p}, \tag{11}$$
$$\bar{A}_{i,1}^{\mathrm{T}}P_i + P_i\bar{A}_{i,1} \leq -\epsilon \mathrm{I}_{n+p}, \tag{12}$$

then the inequality given by

$$\bar{A}_i\big(g_i(t)\big)^{\mathrm{T}}P_i + P_i\bar{A}_i\big(g_i(t)\big) \leq -\epsilon \mathrm{I}_{n+p}, \tag{13}$$

holds for all $g_i(t) \in [0, 1]$, where $\epsilon \in \mathbb{R}_+$. Due to page limitations, the proof is omitted and will be reported elsewhere.

Next, let $z(t) \triangleq [z_1^{\mathrm{T}}(t), z_2^{\mathrm{T}}(t), \ldots, z_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{(n+p)N}$. As a consequence, (9) can be written in the form given by

$$\begin{aligned}\dot{z}(t) &= \begin{bmatrix} \bar{A}_1(t) & & 0 \\ & \ddots & \\ 0 & & \bar{A}_N(t) \end{bmatrix} z(t) \\ &\quad -\alpha \begin{bmatrix} \mathcal{L}_{11}H_1 & \mathcal{L}_{12}H_1 & \ldots & \mathcal{L}_{1N}H_1 \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{L}_{N1}H_N & \mathcal{L}_{N2}H_N & \ldots & \mathcal{L}_{NN}H_N \end{bmatrix} z(t) + \begin{bmatrix} \phi_1(t) \\ \vdots \\ \phi_N(t) \end{bmatrix} \\ &= \underbrace{\begin{bmatrix} \bar{A}_1(t) & & 0 \\ & \ddots & \\ 0 & & \bar{A}_N(t) \end{bmatrix}}_{\bar{A}(t)} z(t) - \alpha \underbrace{\begin{bmatrix} H_1 & & 0 \\ & \ddots & \\ 0 & & H_N \end{bmatrix}}_{H} \\ &\quad \cdot \begin{bmatrix} \mathcal{L}_{11}\mathrm{I}_{n+p} & \mathcal{L}_{12}\mathrm{I}_{n+p} & \ldots & \mathcal{L}_{1N}\mathrm{I}_{n+p} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{L}_{N1}\mathrm{I}_{n+p} & \mathcal{L}_{N2}\mathrm{I}_{n+p} & \ldots & \mathcal{L}_{NN}\mathrm{I}_{n+p} \end{bmatrix} z(t) + \phi(t) \\ &= \bar{A}(t)z(t) - \alpha H(\mathcal{L}(\mathcal{G}) \otimes \mathrm{I}_{n+p})z(t) + \phi(t), \end{aligned} \tag{14}$$

where $\mathcal{L}(\mathcal{G})$ is the Laplacian matrix. In what follows, for each node i, $i = 1, \ldots, N$, we:

i) Solve the linear matrix inequalities given by (11) and (12) for a common positive-definite matrix P_i.

ii) Obtain the design coefficient matrices M_i, S_i, T_i, and N_i from the matrix equality given by

$$H_i = \begin{bmatrix} M_i & S_i \\ T_i & N_i \end{bmatrix} = P_i^{-1}. \tag{15}$$

Notice from (15) that $H = P^{-1}$ (i.e., $PH = \mathrm{I}_N \otimes \mathrm{I}_{n+p} = \mathrm{I}_{N(n+p)}$), where $P \triangleq \mathrm{diag}([P_1, P_2, \ldots, P_N])$. We are now ready to state the main result of this paper. Consider the process given by (1) and the distributed input and state coestimation architecture given by (3) and (4). If there exists a common positive-definite matrix P_i for each node i, $i = 1, \ldots, N$, satisfying (11) and (12), and one selects H_i according to (15), then the sensor network error dynamics given by (14) subject to an undirected and connected graph $\mathcal{G}$ is uniformly bounded. Once again, the proof is omitted due to page limitations and will be reported elsewhere. For interested readers, it follows from the Lyapunov function $V(z) = z^{\mathrm{T}} P z$ as well as the results highlighted in (11), (12) and (13). One can also show that the upper bound on $\|z(t)\|_2$ for $t \geq T$ is proportional to $\mu \triangleq \frac{2\|P\|_2 \bar{\phi}}{\lambda_{\min}(\bar{Q})}$, where $||\phi_i(t)||_2 \leq \bar{\phi}_i$. Since μ depends on the design parameters of the proposed distributed input and state coestimation architecture, it can be used as design metric such that the design parameters can be judiciously selected to make the upper bound on $\|z(t)\|_2$ small.

Finally, we concisely compare the proposed algorithm of this paper with our recent distributed input and state *estimation* approach with time-varying active and passive information roles of nodes [10], which has the form

$$\begin{aligned}\dot{\hat{x}}_i(t) = &(A - \gamma P_i^{-1})\hat{x}_i(t) + B\hat{w}_i(t) \\ &+ g_i(t) L_i\big(y_i(t) - C_i\hat{x}_i(t)\big) \\ &- \alpha P_i^{-1} \sum_{i\sim j} (\hat{x}_i(t) - \hat{x}_j(t)), \quad \hat{x}_i(0) = \hat{x}_{i0}, \quad (16)\end{aligned}$$

$$\begin{aligned}\dot{\hat{w}}_i(t) = &g_i(t) J_i(y_i(t) - C_i\hat{x}_i(t)) - (\sigma_i K_i + \gamma \mathrm{I}_p)\hat{w}_i(t) \\ &- \alpha \sum_{i\sim j} (\hat{w}_i(t) - \hat{w}_j(t)), \quad \hat{w}_i(0) = \hat{w}_{i0}, \quad (17)\end{aligned}$$

with P_i being the positive-definite matrix satisfying

$$R_{i1} \triangleq \begin{bmatrix} A^{\mathrm{T}} P_i + P_i A & -P_i B \\ -B^{\mathrm{T}} P_i & -2\sigma_i K_i \end{bmatrix} \leq 0, \quad (18)$$

$$R_{i2} \triangleq \begin{bmatrix} (A - L_i C_i)^{\mathrm{T}} P_i + P_i (A - L_i C_i) & -P_i B + C_i^{\mathrm{T}} J_i^{\mathrm{T}} \\ -B^{\mathrm{T}} P_i + J_i C_i & -2\sigma_i K_i \end{bmatrix} \leq 0. \quad (19)$$

In particular, this approach has leakage terms in both input and state updates (16) and (17) in the form of "$-\gamma P_i^{-1}\hat{x}_i(t)$" and "$-(\sigma_i K_i + \gamma \mathrm{I}_p)\hat{w}_i(t)$". If the gains "$\gamma P_i^{-1}$" and "$\sigma_i K_i + \gamma \mathrm{I}_p$" in these terms are not small, then they can result in poor performance as well-known. In contrast, the proposed distributed *coestimation* architecture of this paper for heterogeneous sensor networks with time-varying active and passive information roles of nodes has only one leakage term "$-\sigma_i K_i \hat{w}_i(t)$" in the input update (4). Moreover, the proposed architecture of this paper adds the coupling terms "$\hat{x}_i(t) - \hat{x}_j(t)$" and "$\hat{w}_i(t) - \hat{w}_j(t)$" that appear both input and state updates. Finally, the linear matrix inequalities here given by (11) and (12) have a simple structure as compared with the ones in (18) and (19). Owing to these reasons, the proposed coestimation architecture of this paper can be easily (i.e., better) tuned for an overall desired sensor network input and state information fusion performance as opposed to the approach in [10].

IV. Illustrative Numerical Example

To illustrate the proposed distributed input and state coestimation architecture presented and analyzed in Section III, consider the benchmark process [10]–[12] composed of two decoupled systems with the dynamics given by (1), where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_{\mathrm{n1}}^2 & -2\omega_{\mathrm{n1}}\xi_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_{\mathrm{n2}}^2 & -2\omega_{\mathrm{n2}}\xi_2 \end{bmatrix}, \quad (20)$$

$$B = \begin{bmatrix} 0 & 0 \\ \omega_{\mathrm{n1}}^2 & 0 \\ 0 & 0 \\ 0 & \omega_{\mathrm{n2}}^2 \end{bmatrix}, \quad (21)$$

with $\omega_{n1} = 1.2, \xi_1 = 0.9, \omega_{n2} = 1.3$, and $\xi_2 = 0.5$. This process can represent a linearized simple vehicle model with the first and third states corresponding to the positions in the x and y directions, respectively, while the second and fourth states corresponding to the velocities in the x and y directions, respectively. The initial conditions are set to $x_0 = [-2.5,\ 0.5,\ 2.5,\ 0.25]^{\mathrm{T}}$. In addition, we consider the input given by

$$w(t) = \begin{bmatrix} 2.5\sin(0.3t) + 1.5 \\ 1.5\cos(0.5t) \end{bmatrix}. \quad (22)$$

For the numerical results presented in this section, we consider a sensor network with 12 nodes exchanging information over an undirected and connected graph topology as shown in Figure 1, where the active and passive roles of each node are varying with respect to time. We also consider each sensor sensing range to be a circle with the radius $r = 2.5$. If the vehicle's position (the combination of the first and third states) is within a sensor sensing range, then that sensor becomes (smoothly) active. Furthermore, if the vehicle's position is out of the sensor sensing range, then it becomes (smoothly) passive. Note that, for the transition of $g_i(t)$, we use the function $g_i(t) = e^{-\beta t}$ when node i is switching from 1 to 0, and $g_i(t) = 1 - e^{-\beta t}$ when node i is switching from 0 to 1, where β is a positive constant. We adopt this transition from Figure 2(d) of [15].

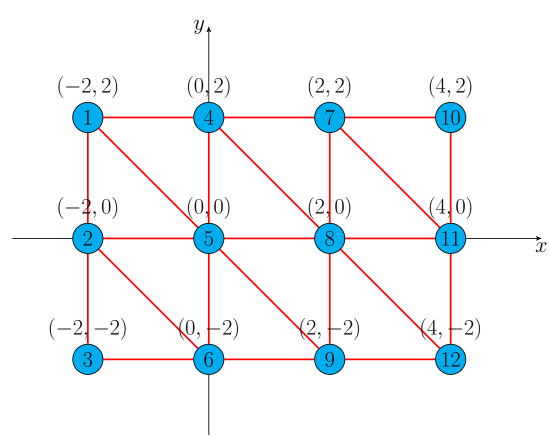

Fig. 1. Communication graph of the time-varying heterogeneous sensor network with 12 nodes (lines denote communication links and circles denote nodes).

Each node's sensing capability is represented by (2) with the output matrices

$$C_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{23}$$

and $\sigma_i = 0.01$ for the odd index nodes and

$$C_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{24}$$

and $\sigma_i = 0.001$ for the even index nodes. The pair (A, C_i) is observable for all $i = 1, \ldots, 12$ in this example; hence, collective observability assumption is satisfied. Moreover, all nodes are subject to zero initial conditions and we set $J_i = \mathrm{diag}([20; 20])$ and $K_i = \mathrm{diag}([10; 10])$ for $i = 1, \ldots, N$. For observer gain L_i, the odd index nodes are subject to

$$L_i = \begin{bmatrix} 20.13 & 0.00 \\ 1.33 & 0.00 \\ 0.00 & 20.32 \\ 0.00 & 3.19 \end{bmatrix}, \tag{25}$$

while the even index nodes are subject to

$$L_i = \begin{bmatrix} -40.17 & 4.14 \\ 57.54 & -5.88 \\ 4.53 & -40.20 \\ -6.45 & 60.42 \end{bmatrix}. \tag{26}$$

For all nodes, we set $\alpha = 25$. In addition, we obtain the common P_i by solving the linear matrix inequalities (11) and (12) with $\epsilon = 0.000001$ for the odd nodes that results in

$$P_1 = \begin{bmatrix} 0.937 & 0.211 & 0.000 & 0.000 & 0.907 & 0.000 \\ 0.211 & 0.333 & 0.000 & 0.000 & 0.191 & 0.000 \\ 0.000 & 0.000 & 0.928 & 0.184 & 0.000 & 0.905 \\ 0.000 & 0.000 & 0.184 & 0.361 & 0.000 & 0.184 \\ 0.907 & 0.191 & 0.000 & 0.000 & 0.986 & 0.000 \\ 0.000 & 0.000 & 0.905 & 0.184 & 0.000 & 1.010 \end{bmatrix}, \tag{27}$$

and $\epsilon = 0.0001$ for the even nodes that results in

$$P_2 = \begin{bmatrix} 1.907 & 1.744 & -0.034 & 0.011 & 1.895 & -0.035 \\ 1.744 & 2.118 & -0.031 & 0.044 & 1.773 & -0.033 \\ -0.034 & -0.031 & 0.862 & 0.649 & -0.036 & 0.856 \\ 0.011 & 0.044 & 0.649 & 1.106 & 0.009 & 0.680 \\ 1.895 & 1.773 & -0.036 & 0.009 & 2.018 & -0.052 \\ -0.035 & -0.033 & 0.856 & 0.680 & -0.052 & 0.980 \end{bmatrix}. \tag{28}$$

That is, $P_1 = P_3 = P_5 = P_7 = P_9 = P_{11}$ and $P_2 = P_4 = P_6 = P_8 = P_{10} = P_{12}$. Based on the matrix P_i, $i = 1, 2, \ldots, 12$, we obtain H_i from (15) and the matrices M_i, S_i, T_i, and N_i are selected accordingly.

For the proposed distributed input and state "coestimation" architecture (3) and (4), sensor network nodes are able to closely estimate the process states and inputs as shown in Figures 2 and 3, respectively. In particular, Figure 4 illustrates that the sensor network is able to estimate the trajectory of the vehicle (the first and third states of the process). For comparison purposes, in addition, we include here the numerical results (see Figures 5, 6 and 7) utilizing the recent architecture in [10] (i.e, utilizing the distributed input and state "estimation" law given by (16) and (17)) for the same scenario outlined above in terms of the dynamics of the process, communication graph of nodes, and

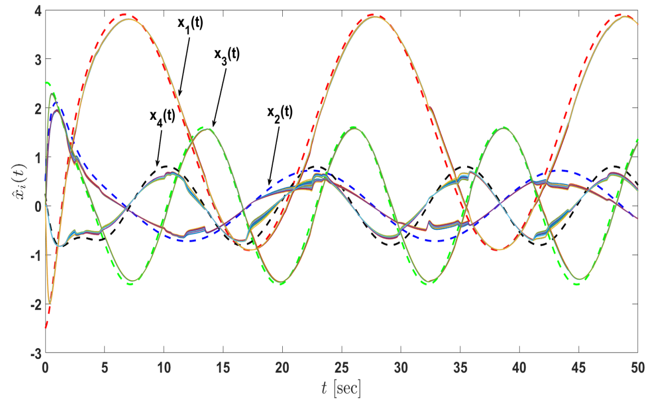

Fig. 2. Time evolution of $\hat{x}_i(t)$, $i = 1, \ldots, N$, of the considered time-varying heterogeneous sensor network under the proposed distributed "coestimation" architecture given by (3) and (4) (the dash lines denote the inputs of the actual process and the solid lines denote the input estimates of nodes).

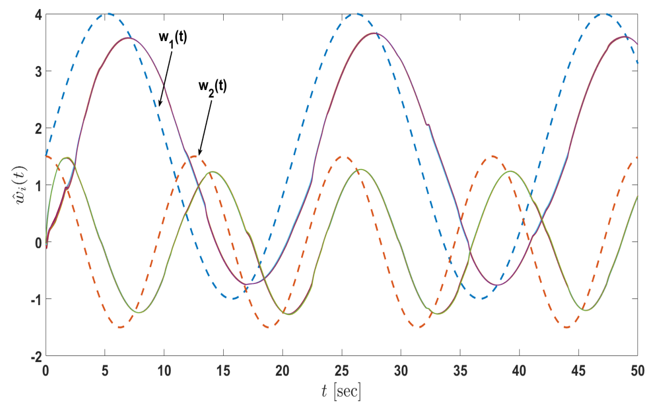

Fig. 3. Time evolution of $\hat{w}_i(t)$, $i = 1, \ldots, N$, of the considered time-varying heterogeneous sensor network under the proposed distributed "coestimation" architecture given by (3) and (4) (the dash lines denote the inputs of the actual process and the solid lines denote the input estimates of nodes).

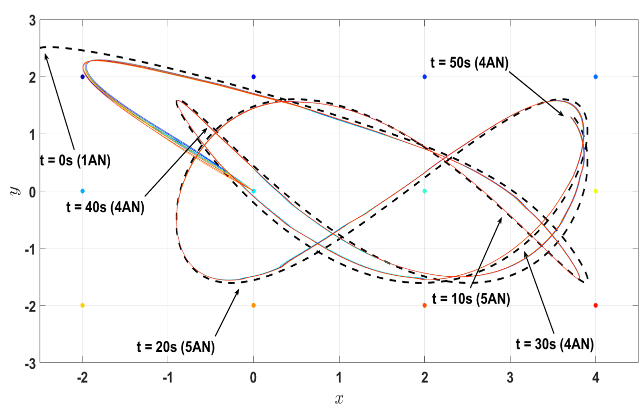

Fig. 4. Position estimates (first and third states of the process) of the considered time-varying heterogeneous sensor network under the proposed distributed "coestimation" architecture given by (3) and (4) (the dash line denotes the trajectory of the actual process (i.e. the combination of the first and third state) and the solid lines denote the state estimates of nodes). Here, AN stands for the the active nodes.

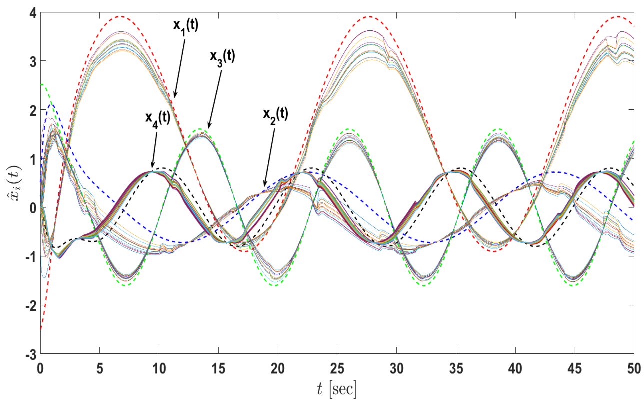

Fig. 5. Time evolution of $\hat{x}_i(t)$, $i = 1, \ldots, N$, of the considered time-varying heterogeneous sensor network under the recent distributed "estimation" architecture in [10] given by (16) and (17) (the dash lines denote the inputs of the actual process and the solid lines denote the input estimates of nodes).

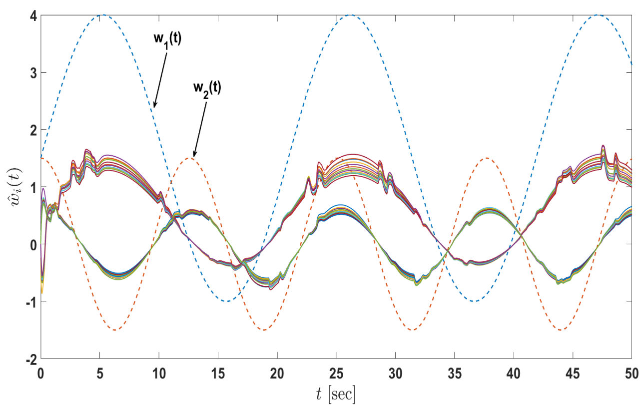

Fig. 6. Time evolution of $\hat{w}_i(t)$, $i = 1, \ldots, N$, of the considered time-varying heterogeneous sensor network under the recent distributed "estimation" architecture in [10] given by (16) and (17) (the dash lines denote the inputs of the actual process and the solid lines denote the input estimates of nodes).

sensors' modalities. To summarize, this comparison study clearly highlights the substantially improved dynamic input and state fusion performance of the proposed distributed "coestimation" architecture of this paper versus our recent distributed "estimation" approach in [10].

V. Conclusion

In order to contribute to the previous studies in distributed algorithm synthesis and analysis for sensor networks, we addressed the problem of system-theoretic dynamic information fusion in time-varying heterogeneous sensor networks, which involve time-varying set of active and passive information roles subject to nonidentical modalities. To this end, a new distributed input and state "coestimation" architecture was proposed, where the key feature of our framework was that time evolution of input and state updates of each node both depend on the local input and state information exchanges. As compared with our recent distributed input and state "estimation" approach documented in [10] for the same problem, where time evolution of input (respectively, state) update of each node only depends the local input (respectively, state) information exchange, our illustrative numerical example

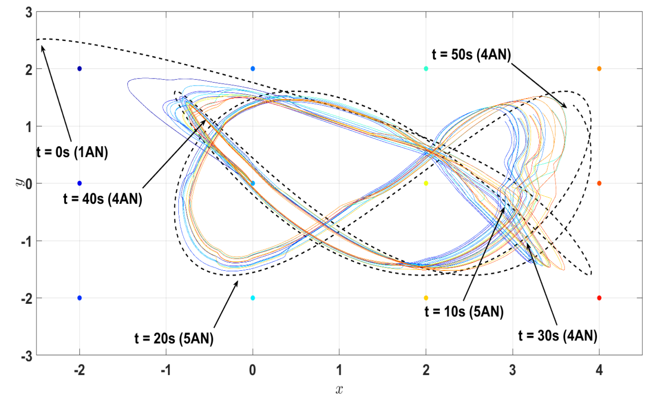

Fig. 7. Position estimates (first and third states of the process) of the considered time-varying heterogeneous sensor network under the reent distributed "estimation" architecture in [10] given by (16) and (17) (the dash line denotes the trajectory of the actual process (i.e. the combination of the first and third state) and the solid lines denote the state estimates of nodes). Here, AN stands for the the active nodes.

also demonstrated a substantially improved dynamic input and state fusion performance.

References

[1] T. Yucelen and J. D. Peterson, "Distributed control of active-passive networked multiagent systems," *IEEE Conference on Decision and Control*, 2014.

[2] J. D. Peterson, T. Yucelen, G. Chowdhary, and S. Kannan, "Exploitation of heterogeneity in distributed sensing: An active-passive networked multiagent systems approach," in *American Control Conference*, 2015, pp. 4112–4117.

[3] D. W. Casbeer, Y. Cao, E. Garcia, and D. Milutinović, "Average bridge consensus: Dealing with active-passive sensors," in *ASME Dynamic Systems and Control Conference*, 2015.

[4] J. D. Peterson, T. Yucelen, and E. Pasiliao, "Generalizations on active-passive dynamic consensus filters," in *American Control Conference*, 2016, pp. 3740–3745.

[5] Y. Cao, D. Casbeer, E. Garcia, and X. Zhang, "Towards cost-effective distributed information fusion with partially active sensors in directed networks," in *IEEE Conference on Decision and Control*, 2016, pp. 4308–4313.

[6] T. Yucelen and J. D. Peterson, "Distributed control of active-passive networked multiagent systems," in *IEEE Transactions on Control of Network Systems*, 2016.

[7] S. Mou, E. Garcia, and D. Casbeer, "Distributed algorithms for the average bridge consensus," in *IEEE Conference on Control Technology and Applications*, 2017.

[8] R. Olfati-Saber, "Distributed Kalman filtering for sensor networks," in *IEEE Conference on Decision and Control*, 2007, pp. 5492–5498.

[9] P. Millán, L. Orihuela, C. Vivas, F. Rubio, D. V. Dimarogonas, and K. H. Johansson, "Sensor-network-based robust distributed control and estimation," *Control Engineering Practice*, vol. 21, no. 9, pp. 1238–1249, 2013.

[10] D. Tran, T. Yucelen, S. B. Sarsilmaz, and S. Jagannathan, "Distributed input and state estimation using local information in heterogeneous sensor networks," *Frontiers in Robotics and AI*, vol. 4, p. 30, 2017.

[11] D. Tran, T. Yucelen, and S. Jagannathan, "On local design and execution of a distributed input and state estimation architecture for heterogeneous sensor networks," in *American Control Conference*, 2017, pp. 3874–3879.

[12] ——, "A new result on distributed input and state estimation for heterogeneous sensor networks," in *ASME Dynamic Systems and Control Conference*, 2017.

[13] M. Mesbahi and M. Egerstedt, *Graph theoretic methods in multiagent networks*. Princeton University Press, 2010.

[14] C. D. Godsil, G. Royle, and C. Godsil, *Algebraic graph theory*. Springer New York, 2001, vol. 207.

[15] M. M. Zavlanos, M. B. Egerstedt, and G. J. Pappas, "Graph-theoretic connectivity control of mobile robot networks," *Proceedings of the IEEE*, vol. 99, no. 9, pp. 1525–1540, 2011.